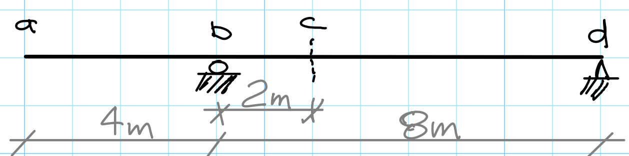
Influence Lines

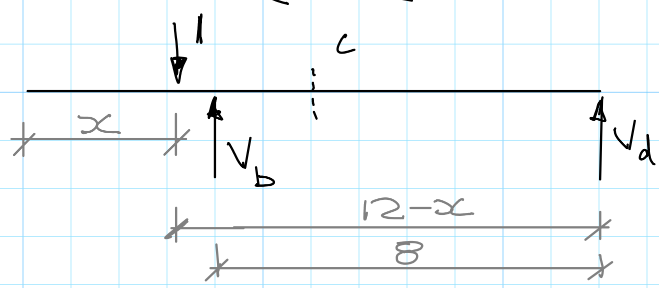
Influence lines plot the effect of a single moving load at a specified point in a structure. In contrast, shear force and bending moment diagrams plot the effect of a static set of loads on all points of the structure.

For example, construct the influence lines for moment & for shear at pt c in the following structure. In other worlds, plot how these two effects vary as a unit force moves from one end (a) to the other (d).



i) place unit load to the left of pt. c

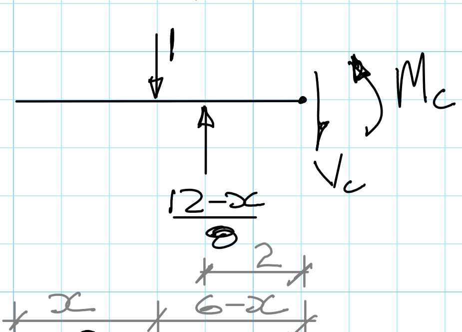
$0 \le x \le 6\,m$



$\Sigma M_d = 0$ (+↺)

$1(12-x) - V_b(8) = 0$

$V_b = \frac{12-x}{8}$



$\Sigma M_c = 0$ (+↺)

$M_c + 1(6-x) - \left(\frac{12-x}{8}\right) \times 2 = 0$

$M_c + 6 - x - 3 + \frac{x}{4} = 0$

$M_c = \frac{3x}{4} - 3$

@ $x = 0$ $\quad M_c = -3$

@ $x = 6$ $\quad M_c = 1.5$

$\Sigma F_y = 0$ (+↑)

$-1 + \frac{12-x}{8} - V_c = 0$

$V_c = 0.5 - \frac{x}{8}$

$0 \le x \le 6$

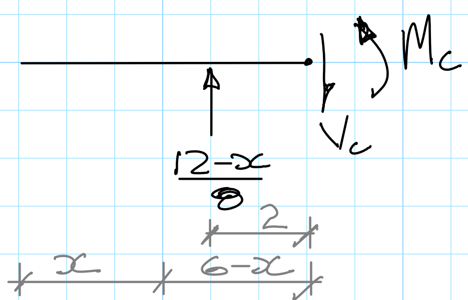
@ $x = 0$ $\quad V_c = 0.5$

@ $x = 6$ $\quad V_c = -0.25$

ii) now place unit load to the right of c
$6 \le x \le 12$

- the overall FBD is unchanged so the left reaction is the same

$$V_b = \frac{12-x}{8}$$



$\Sigma F_y = 0 \; +\uparrow$

$$V_c = \frac{12-x}{8}$$

$$= 1.5 - \frac{x}{8}$$

@ $x = 6$ $V_c = 0.75$
@ $x = 12$ $V_c = 0$

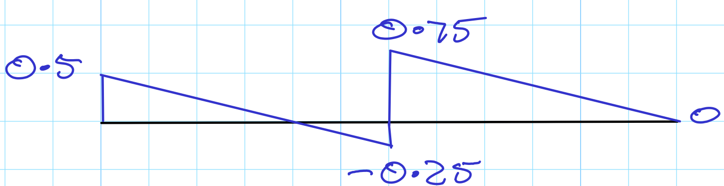
$\Sigma M_c = 0$ (+↻)

$$M_c - 2 \times \frac{12-x}{8} = 0$$

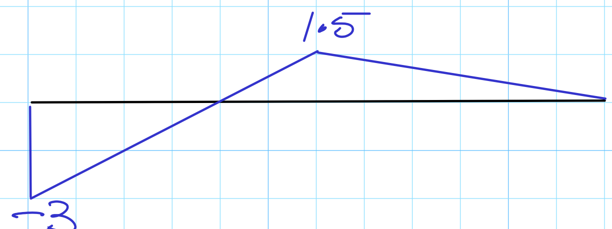
$$M_c = 3 - \frac{x}{4}$$

@ $x = 6$ $M_c = 1.5$
@ $x = 12$ $M_c = 0$

iii) Plot



I.L. for shear @ c



I.L. for moment @ c

Use of Influence Lines

1) Determine the effects @ c due to a single 30 kN force that can be placed anywhere on the span.

By inspection, max shear @ c occurs when load is placed just to the right of c.

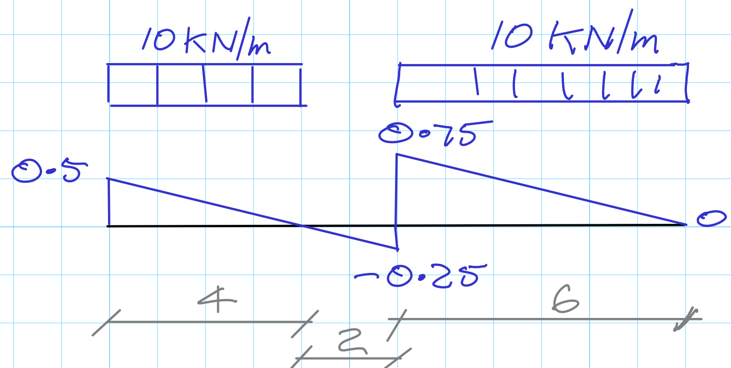
$$V_{c_{max}} = 0.75 \times 30 = 22.5 \text{ kN.}$$

max moment @ c occurs when load is at pt. a.

$$M_{c_{max}} = -3 \times 30 = -90 \text{ kN-m.}$$

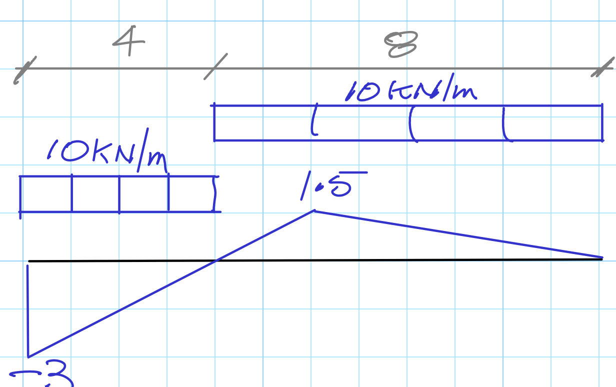
2) Determine max effects @ c due to 10 kN/m live load.

- place loads over all regions of I.L. that have the same sign.
- the resulting effect = load × area of I.L.



$$V_c = 10 \times 0.5 \times 4 \times \tfrac{1}{2} + 10 \times 0.75 \times 6 \times \tfrac{1}{2}$$

$$V_{c_{max}} = 32.5 \text{ kN.}$$



$$M_c^- = 10 \times -3 \times 4 \times \tfrac{1}{2} = -60 \text{ kN-m}$$

$$M_c^+ = 10 \times 8 \times 1.5 \times \tfrac{1}{2} = +60 \text{ kN-m}$$

$$M_{c_{max}} = \pm 60 \text{ kN-m}$$