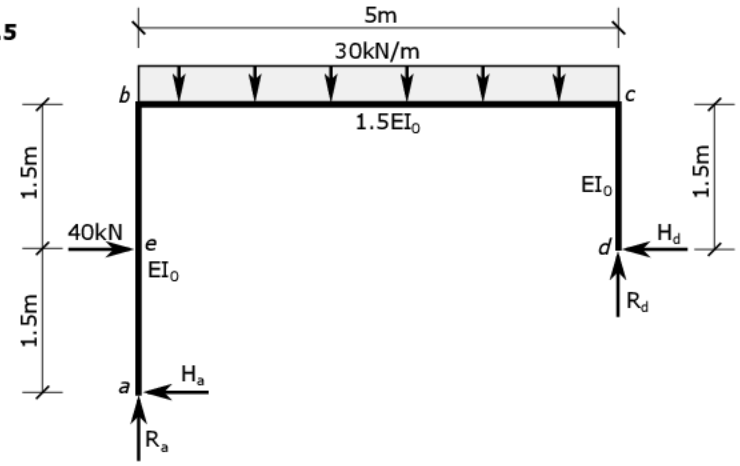
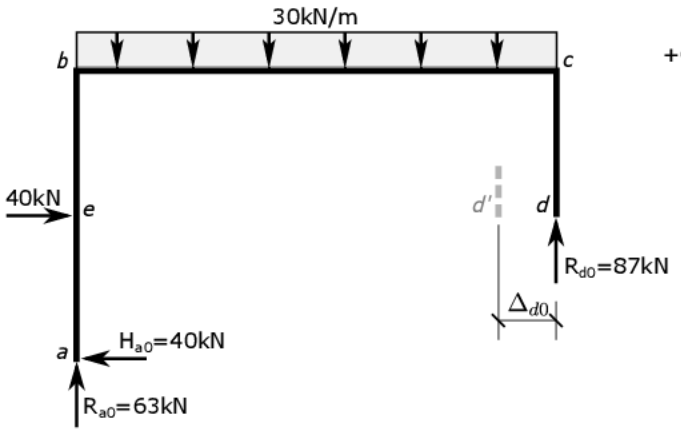
Problem 8.5

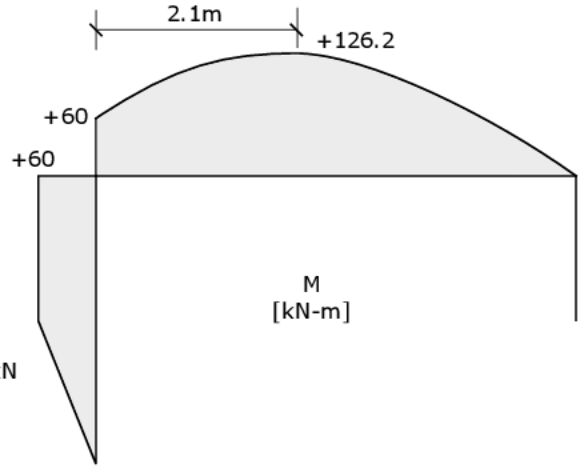
Real Structure



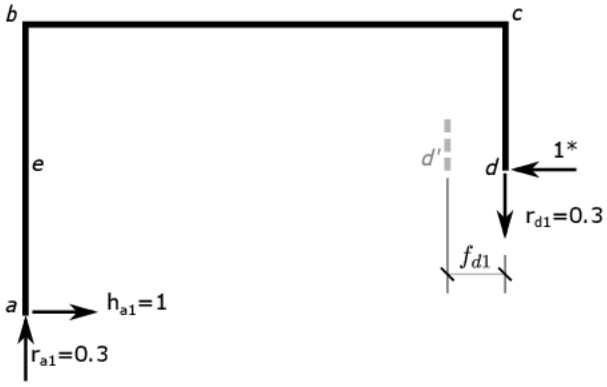
1) 1° S.I. Choose H_d as the redundant.

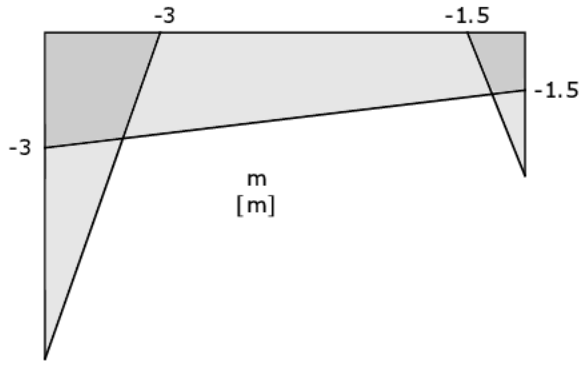
Primary Structure





Unit Value of Redundant





Displacement in primary structure:

$$\Delta_{d0} = \int \frac{mM}{EI}$$

$$= \frac{1}{EI_o}\int \begin{matrix}1.5\\-1.5\end{matrix} \quad 60 \qquad \text{(segment a-e)}$$

$$+ \frac{1}{EI_o}\int \begin{matrix}1.5\\-1.5\end{matrix} \quad 60 \quad \begin{matrix}\\-3\end{matrix} \qquad \text{(segment e-b)}$$

$$+ \frac{1}{1.5EI_o}\int \begin{matrix}5\\-3\end{matrix} \quad 60 \quad \begin{matrix}30\\0\end{matrix} \qquad \text{(segment b-c)}$$

$$= \frac{1}{EI_o}\left[\frac{1.5}{3}\times -1.5\times 60 + \frac{1.5}{2}(-1.5+-3)60 + \frac{1}{1.5}\times\frac{5}{24}\left[(-3)(30\times 5^2+8\times 60)+(-1.5)(30\times 5^2+4(60))\right]\right]$$

$$\Delta_{d0} = \frac{-1168.75\ \text{kN-m}^3}{EI_o} \quad (\therefore \leftarrow)$$

Flexibility Coefficient (displ. due to unit value of redundant)

$$f_{d1} = \int \frac{mm}{EI}$$

$$= \frac{1}{EI_o}\int \left(\begin{matrix}3\\-3\end{matrix}\right)^2 \qquad \text{(seg. a-b)}$$

$$+ \frac{1}{1.5EI_o}\int \left(\begin{matrix}5\\-1.5\end{matrix}\right)^2 \qquad \text{(seg b-c)}$$

$$+ \frac{1}{EI_o}\int \left(\begin{matrix}1.5\\-1.5\end{matrix}\right)^2 \qquad \text{(seg c-d)}$$

$$= \frac{1}{EI_o}\left[\frac{3}{3}\times -3\times -3 + \frac{1}{1.5}\frac{5}{6}\left[-3(2\times -3+-1.5)+-1.5(-3+2\times -1.5)\right] + \frac{1.5}{3}\times -1.5\times -1.5\right]$$

$$= \frac{27.625\ \text{m}^3}{EI_o}$$

Compatibility:

$$0 = \Delta_{d0} + H_d f_{d1}$$

$$= \frac{-1168.75\ \text{kNm}^3}{EI_o} + H_d\frac{27.625\ \text{m}^3}{EI_o}$$

Solve:

$$H_d = 42.31\ \text{kN} \quad (\therefore \leftarrow)$$

Superposition to find reactions & moments:

$H_a = H_{a0} - H_d h_{a1}$
$= 40 - 42.31 \times 1$
$= -2.31$ kN $(\leftarrow)$

$R_a = R_{a0} + H_d r_{a1}$
$= 63 + 0.3 \times 42.31$
$= 75.7$ kN $(\uparrow)$

$R_d = R_{d0} - H_d r_{a1}$
$= 87 - 0.3 \times 42.31$
$= 74.3$ kN $(\uparrow)$

$M_b = 60 - 3 \times 42.31$
$= -66.9$ kN-m $(\circlearrowright)$

$M_c = 0 - 1.5 \times 42.31$
$= -63.5$ kN-m $(\circlearrowleft)$