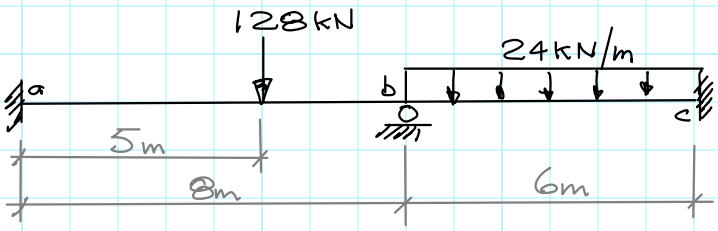
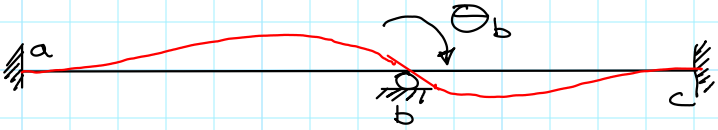
Beam Example 1 (In Class Nov 28, 30)
Method of Slope Deflection



EI constant

Step 1) Identify & label the independent joint displacements (DOF) - these must be solved for



1 D.O.F.

Step 2) Determine the fixed-end moments; these are the moments that would be in each member end if they were all fixed against displacement (rotation, in this case).

$$M^f_{ab} = -\frac{Pab^2}{L^2} = -\frac{128\text{kN} \times 5\text{m} \times (3\text{m})^2}{(8\text{m})^2}$$

$$= -90\text{kN-m}$$

$$M^f_{ba} = \frac{Pa^2b}{L^2} = \frac{128\text{kN} \times (5\text{m})^2 \times 3\text{m}}{(8\text{m})^2}$$

$$= +150\text{kN-m}$$

$$M^f_{bc} = -\frac{wL^2}{12} = \frac{24\text{kN/m} \times (6\text{m})^2}{12}$$

$$= -72\text{kN-m}$$

$$M^f_{cb} = \frac{wL^2}{12} = \frac{24\text{kN/m} \times (6\text{m})^2}{12}$$

$$= +72\text{kN-m}$$

Step 3) Write slope-deflection equations. These express moments at the ends of each segment in terms of the unknown joint displacements identified in step 1).

$$M_{ab} = \frac{EI}{L}\left(4\theta_a + 2\theta_b - 6\frac{\Delta_{ab}}{L}\right) + M^f_{ab}$$

$$= \frac{EI}{8m}\left(2\theta_b\right) - 90\,kN\text{-}m$$

$$= \frac{EI}{4m}\theta_b - 90\,kN\text{-}m$$

$$M_{ba} = \frac{EI}{L}\left(2\theta_a + 4\theta_b - 6\frac{\Delta_{ab}}{L}\right) + M^f_{ba}$$

$$= \frac{EI}{8m}\left(4\theta_b\right) + 150\,kN\text{-}m$$

$$= \frac{EI}{2m}\theta_b + 150\,kN\text{-}m$$

similarly

$$M_{bc} = \frac{EI}{6m}\left(4\theta_b\right) - 72\,kN\text{-}m$$

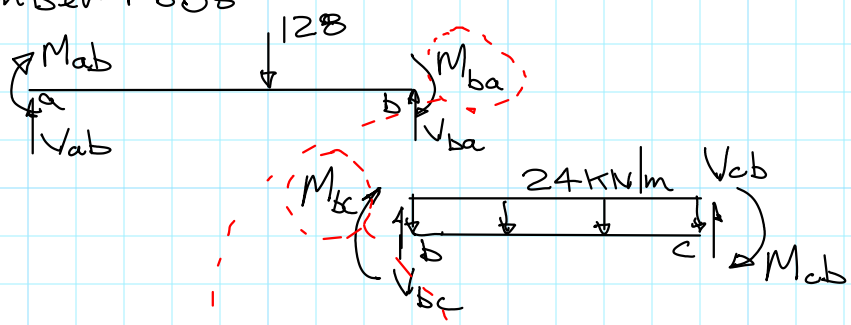
$$= \frac{2EI}{3m}\theta_b - 72\,kN\text{-}m$$

$$M_{cb} = \frac{EI}{6m}\left(2\theta_b\right) + 72\,kN\text{-}m$$

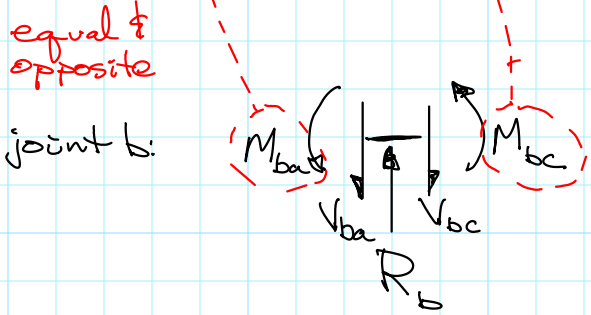
$$= \frac{EI}{3m}\theta_b + 72\,kN\text{-}m$$

Step 4) Write equilibrium equations involving the end moments (and therefore also the unknown displacements)

member FBDs:



joint FBD's (forces are equal & opposite to member end forces)



$$\Sigma M_b = 0 \quad (+\circlearrowleft)$$

$$M_{ba} + M_{bc} = 0$$

$$\frac{EI}{2m}\theta_b + 150\,kN\text{-}m + \frac{2EI}{3m}\theta_b - 72\,kN\text{-}m = 0$$

$$\frac{EI}{m}\left(\frac{1}{2} + \frac{2}{3}\right)\theta_b + 78\,kN\text{-}m = 0$$

Step 5) Solve for the joint displacements

$$\theta_b = \frac{-78\,kN\text{-}m^2}{EI} \times \left(\frac{1}{\frac{1}{2} + \frac{2}{3}}\right)$$

$$\theta_b = \frac{-468\,kN\text{-}m^2}{7EI} \qquad (\therefore \circlearrowleft)$$

Step 6) Back substitute displacements into slope deflection equations to obtain member end moments.

$$M_{ab} = \frac{EI}{4m} \times -\frac{468\ kN\text{-}m^2}{7\ EI} - 90\ kN\text{-}m = -106.7\ kN\text{-}m \quad (\circlearrowleft)$$

$$M_{ba} = \frac{EI}{2m} \times -\frac{468\ kN\text{-}m^2}{7EI} + 150\ kN\text{-}m = 116.6\ kN\text{-}m \quad (\circlearrowright)$$

$$M_{bc} = \frac{2EI}{3m} \times -\frac{468\ kN\text{-}m^2}{7\ EI} - 72\ kN\text{-}m = -116.6\ kN\text{-}m \quad (\circlearrowleft)$$

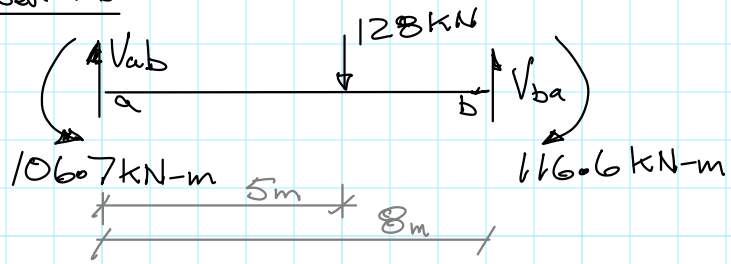
$$M_{cb} = \frac{EI}{3m} \times -\frac{468\ kN\text{-}m^2}{7EI} + 72\ kN\text{-}m = +49.7\ kN\text{-}m \quad (\circlearrowright)$$

we observe that $M_{ba} + M_{bc} = 0$, as it should. this is a check that we solved the eqns properly. It does not ensure that the eqns were correct - that comes later.

Step 7) Determine member end shears & reactions using ~~statics~~.

Draw FBD of each member.

member a-b



$$\Sigma M_a = 0 \quad (+\circlearrowleft)$$

$$-128 \times 5 - 116.6 + 106.7 + V_{ba}(8) = 0$$

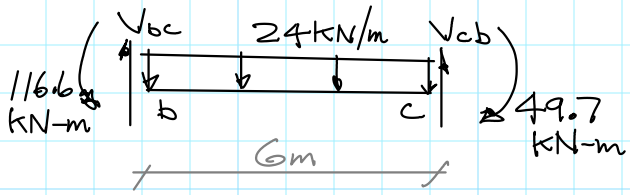
$$V_{ba} = 81.2 \quad (\uparrow)$$

$$\Sigma F_y = 0 \quad +\uparrow$$

$$V_{ab} - 128 + 81.2 = 0$$

$$V_{ab} = 46.8 \quad (\uparrow)$$

member bc



$\Sigma M_b = 0 \quad (+\circlearrowleft)$

$-24\frac{kN}{m} \times 6m \times 3m + V_{cb}(6m) - 49.7\,kN\text{-}m + 116.6\,kN\text{-}m = 0$

$V_{cb} = 60.8\,kN \quad (\therefore \uparrow)$

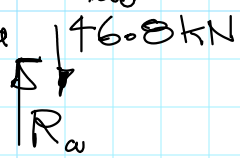
$\Sigma F_y = 0 \quad +\uparrow$

$V_{bc} - 24\frac{kN}{m} \times 6m + 60.8\,kN = 0$

$V_{bc} = 83.2\,kN \quad (\therefore \uparrow)$

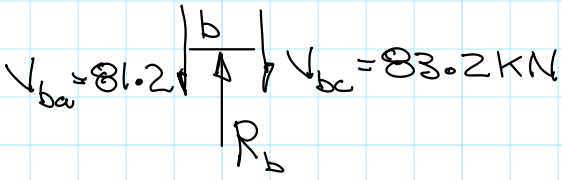
Now from joint FBD's (observing equal & opposite nature of shears) obtain the reactions.

joint a:



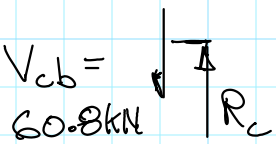
$R_a = 46.8\,kN$

joint b:



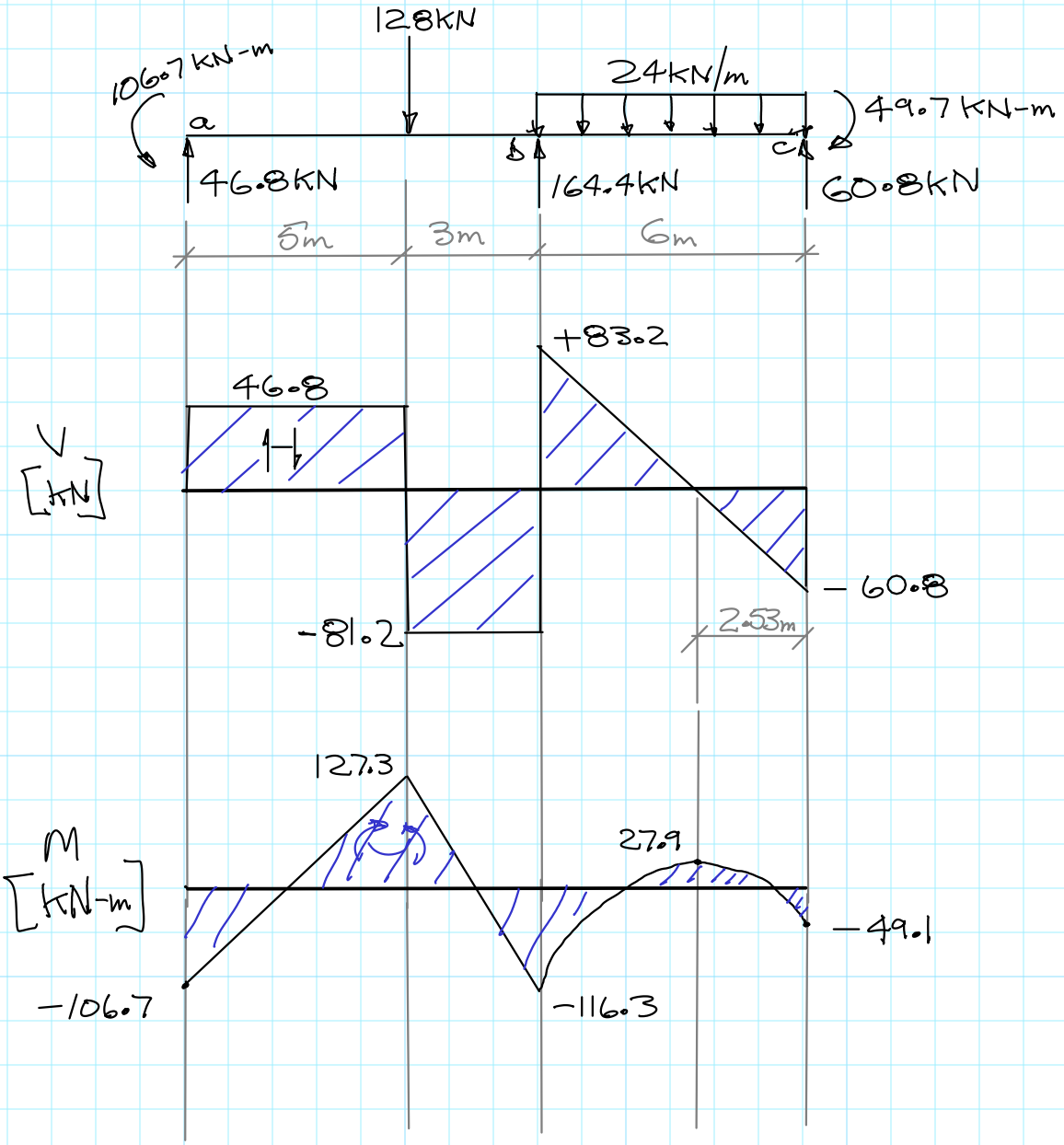
$R_b = 81.2\,kN + 83.2\,kN = 164.4\,kN$

joint c:



$R_c = 60.8\,kN$

Step 8) Complete FBD, V & M diagrams



The values on the M diagram were determined by integrating the V diag. Compare with those compute in Step 6)

M_{cb} = 49.7 there
49.1 here Ok.