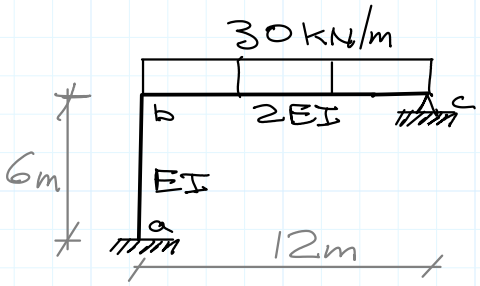
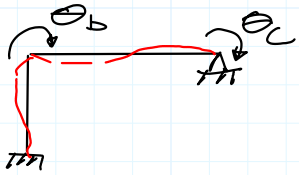
Example 2 – Determine member end forces and reactions



1) determine DOF



2) fixed end moments

$$M^F_{ab} = M^F_{ba} = 0$$

$$M^F_{bc} = -\frac{30 \times 12^2}{12} = -360 \text{ kN-m}$$

$$M^F_{cb} = +360 \text{ kN-m}$$

3) s-d eqns

$$M_{ab} = \frac{EI}{6m} \times 2\theta_b = \frac{EI}{3m}\theta_b$$

$$M_{ba} = \frac{EI}{6m} \times 4\theta_b = \frac{2EI}{3m}\theta_b$$

$$M_{bc} = \frac{2EI}{12m}(4\theta_b + 2\theta_c) - 360 \text{kN-m}$$

$$= \frac{2EI}{3m}\theta_b + \frac{EI}{3m}\theta_c - 360 \text{ kN-m}$$

$$M_{cb} = \frac{2EI}{12m}(4\theta_c + 2\theta_b) + 360 \text{ kN-m}$$

$$= \frac{EI}{3m}\theta_b + \frac{2EI}{3m}\theta_c + 360 \text{ kN-m.}$$

4) equilibrium eqns (2 unknowns ∴ 2 reqd)

@ joint b $\quad M_{ba} + M_{bc} = 0$

$$\frac{2EI}{3m}\theta_b + \frac{2EI}{3m}\theta_b + \frac{EI}{3m}\theta_c - 360 = 0$$

$$\frac{4EI}{3m}\theta_b + \frac{EI}{3m}\theta_c = 360 \text{ kN-m}$$

@ joint c $\quad M_{cb} = 0$

$$\frac{EI}{3m}\theta_b + \frac{2EI}{3m}\theta_c = -360 \text{ kN-m}$$

5) solving

$$\frac{EI}{3m}\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \theta_b \\ \theta_c \end{Bmatrix} = \begin{Bmatrix} 360 \\ -360 \end{Bmatrix} \text{kN-m}$$

$$\begin{Bmatrix} \theta_b \\ \theta_c \end{Bmatrix} = \frac{3m}{EI}\begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}\frac{1}{4\times2-1\times1}\begin{Bmatrix} 360 \\ -360 \end{Bmatrix} \text{kN-m}$$

$$\begin{Bmatrix} \theta_b \\ \theta_c \end{Bmatrix} = \begin{Bmatrix} \frac{3240}{7} \\ \frac{-5400}{7} \end{Bmatrix}\frac{\text{kN-m}^2}{EI}$$

6) back substitute in s-d eqns

$$M_{ab} = \frac{EI}{3m} \times \frac{3240}{7}\frac{\text{kN-m}^2}{EI} = +154.3 \text{ kN-m}$$

$$M_{ba} = \frac{2EI}{3m} \times \frac{3240}{7}\frac{\text{kN-m}^2}{EI} = +308.6 \text{ kN-m}$$

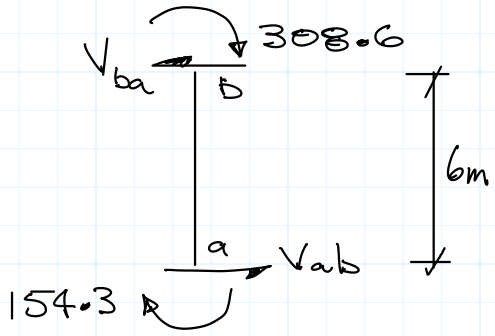
$$M_{bc} = \frac{2EI}{3m} \times \frac{3240}{7}\frac{\text{kN-m}^2}{EI} + \frac{EI}{3m} \times \frac{-5400}{7}\frac{\text{kN-m}^2}{EI} - 360 \text{ kN-m}$$

$$= -308.6 \text{ kN-m} \quad \checkmark\checkmark$$

$$M_{cb} = \frac{EI}{3m} \times \frac{3240}{7}\frac{\text{kN-m}^2}{EI} + \frac{2EI}{3m} \times \frac{-5400}{7}\frac{\text{kN-m}^2}{EI} + 360 \text{ kN-m}$$

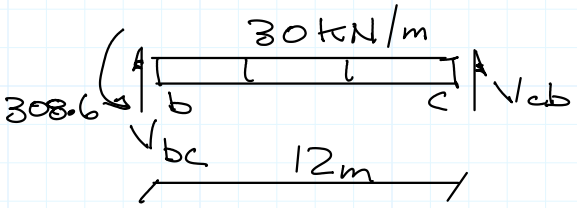
$$= 0 \quad \checkmark\checkmark$$

7) member end shears



$$V_{ab} = \frac{308.6 + 154.3}{6} = 77.2 \text{ kN}$$

$$V_{ba} = 77.2 \text{ kN}$$



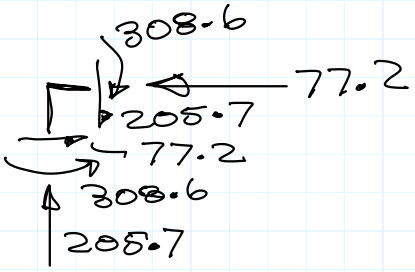
$$V_{cb} = \frac{30 \times 12 \times 6 - 308.6}{12}$$

$$= 154.3 \text{ kN}$$

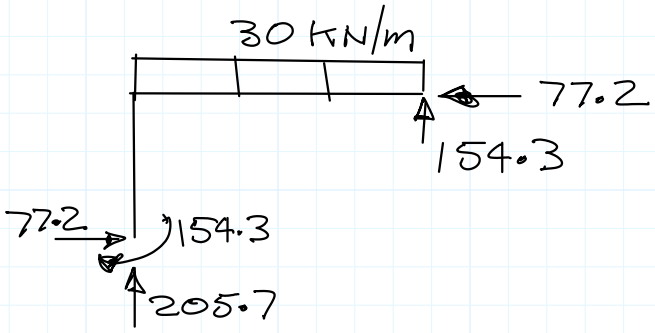
$$V_{bc} = 30 \times 12 - 154.3$$

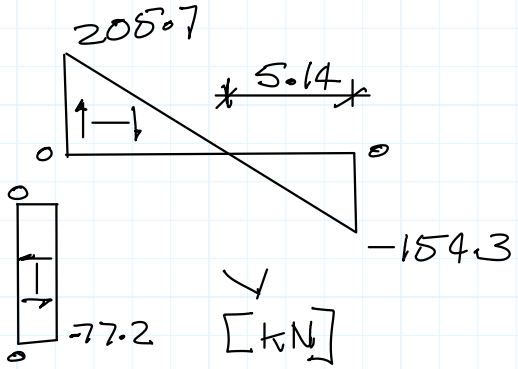
$$= 205.7 \text{ kN}$$

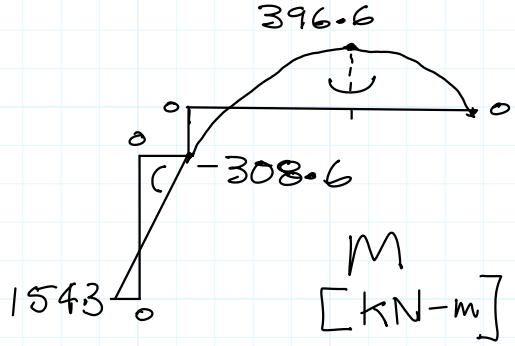
joint b



Summary







Note: it is possible to use a modified version of the FEMs & s-d eqns recognizing that end c of bc is pinned - doing so requires only 1 DOF - θ_b. Reduces the work slightly. See next example.