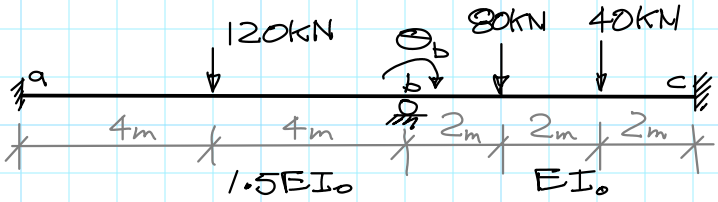
Problem 10.2.6 -1

1 D.O.F. (θ_b)



Fixed End Moments

$$M^f_{ab} = -\frac{120\,kN \times 8m}{8} = -120\,kN\text{-}m$$

$$M^f_{ba} = +120\,kN\text{-}m$$

$$M^f_{bc} = -\frac{80 \times 2 \times 4^2}{6^2} + -\frac{40 \times 4 \times 2^2}{6^2} = -88.89\,kN\text{-}m$$

$$M^f_{cb} = \frac{80 \times 2^2 \times 4}{6^2} + \frac{40 \times 4^2 \times 2}{6^2} = 71.11\,kN\text{-}m$$

S.D. Eqns

$$M_{ab} = \frac{1.5EI_o}{8m}\left(4\cancel{\theta_a}^{\,0} + 2\theta_b - 6\cancel{\frac{\Delta_{ab}}{L}}^{\,0}\right) + M^f_{ab}$$

$$= \frac{3EI_o}{8m}\theta_b - 120\,kN\text{-}m$$

$$M_{ba} = \frac{1.5EI_o}{8m}\left(4\theta_b\right) + M^f_{ba}$$

$$= \frac{3EI_o}{4m}\theta_b + 120\,kN\text{-}m$$

$$M_{bc} = \frac{EI_o}{6m}\left(4\theta_b\right) + M^f_{bc}$$

$$= \frac{2EI_o}{3m}\theta_b - 88.89\,kN\text{-}m$$

$$M_{cb} = \frac{EI_o}{6m}\left(2\theta_b\right) + M^f_{cb}$$

$$= \frac{EI_o}{3m}\theta_b + 71.11\,kN\text{-}m$$

Equilibrium

$$M_{ba} + M_{bc} = 0$$

$$\frac{3EI_o}{4m}\theta_b + 120\,kN\text{-}m + \frac{2EI_o}{3m}\theta_b - 88.89\,kN\text{-}m = 0$$

Solve

$$\frac{17EI_o}{12m}\theta_b = -31.11\,kN\text{-}m$$

$$\theta_b = \frac{-21.96\,kN\text{-}m^2}{EI_o} \quad (\circlearrowleft)$$

Back Sub.

$$M_{ab} = \frac{3EI_o}{8m}\theta_b - 120\,kN\text{-}m$$

$$= \frac{3}{8m} \times -21.96\,kN\text{-}m^2 - 120\,kN\text{-}m$$

$$= -128\,kN\text{-}m \quad (\therefore \circlearrowleft)$$

$$M_{ba} = \frac{3}{4m} \times -21.96\,kN\text{-}m^2 + 120\,kN\text{-}m$$

$$= +103.5\,kN\text{-}m \quad (\therefore \circlearrowright)$$

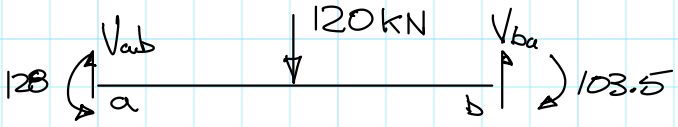
$$M_{bc} = \frac{2}{3m} \times -21.96\,kN\text{-}m^2 - 88.89\,kN\text{-}m$$

$$= -103.5\,kN\text{-}m \quad (\therefore \circlearrowleft)$$

$$M_{cb} = \frac{1}{3m} \times -21.96\,kN\text{-}m^2 + 71.11\,kN\text{-}m$$

$$= 63.8\,kNm \quad (\therefore \circlearrowright)$$

Member end shears

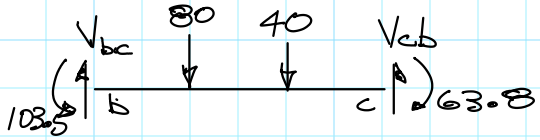


$$\Sigma M_b = 0 \quad (+\circlearrowleft)$$

$$128 - V_{ab}(8) + 120(4) - 103.5 = 0$$

$$V_{ab} = 63.06 \text{ kN}$$

$$V_{ba} = 120 - 63.06 = 56.94 \text{ kN}$$

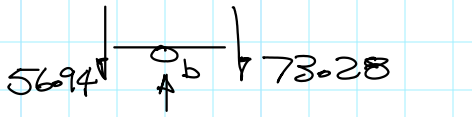


$$V_{bc} = \frac{1}{6}(103.5 - 63.8 + 80\times4 + 40\times2) \qquad (\Sigma M_c)$$

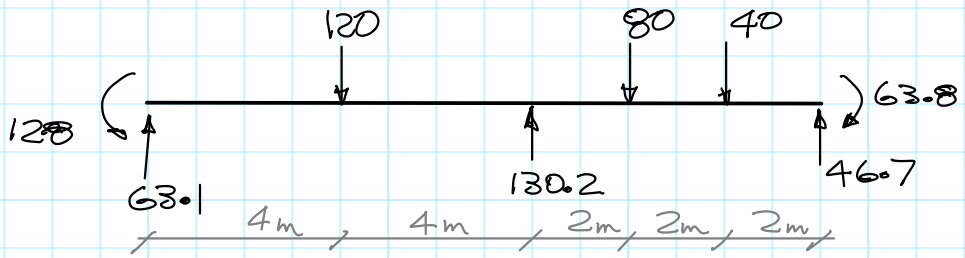
$$= 73.28 \text{ kN}$$

$$V_{cb} = 80 + 40 - 73.28 = 46.72$$

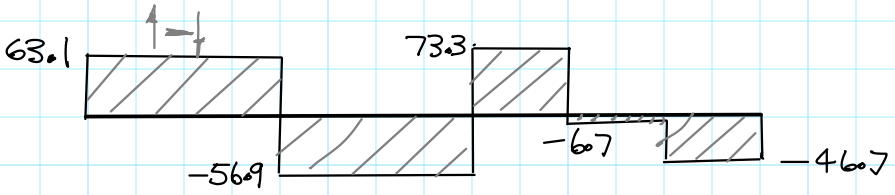
Reactions



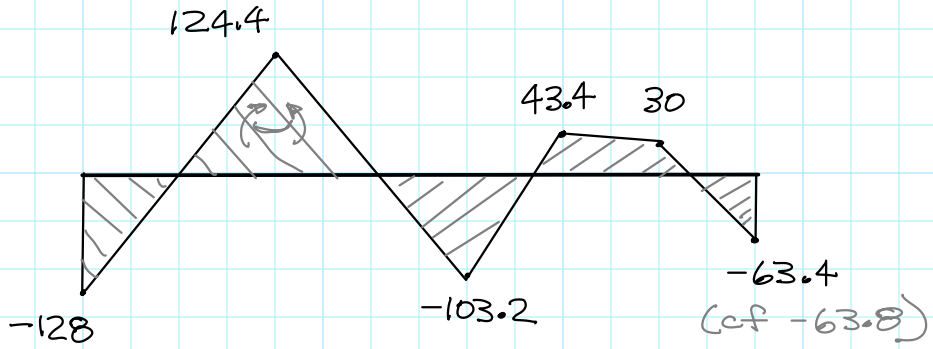
$$R_b = 130.2 \text{ kN}$$



FBD



V
[kN]



M
[kN-m]

Note – V values come from integrating FBD & M values come from integrating V diag – this is an equilibrium check on whole beam.

OK.