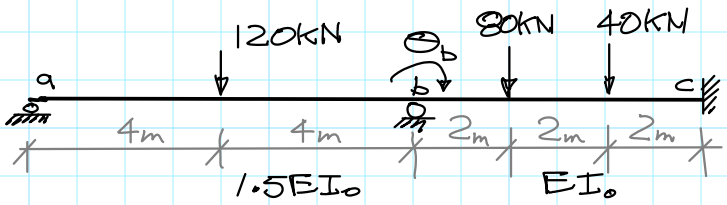
Problem 10.2.6 -2



1 D.O.F. (θ_b)
[Moment is known to be 0 @ a; therefore we can use modified SD in ab & θ_a need not be determined]

Fixed End Moments

$$M^f_{ab} = 0$$

$$M^f_{ba} = \frac{3 \times 120\,kN \times 8m}{16} = 180\,kN\text{-}m$$

$$M^f_{bc} = -\frac{80 \times 2 \times 4^2}{6^2} + -\frac{40 \times 4 \times 2^2}{6^2} = -88.89\,kN\text{-}m$$

$$M^f_{cb} = \frac{80 \times 2^2 \times 4}{6^2} + \frac{40 \times 4^2 \times 2}{6^2} = 71.11\,kN\text{-}m$$

S.D. Eqns

$$M_{ab} = 0$$

$$M_{ba} = \frac{1.5EI_0}{8m}\left(3\theta_b - \frac{3\Delta_{ab}}{L}\right) + M^f_{ba}$$ (0)

$$= \frac{9EI_0}{16m}\theta_b + 180\,kN\text{-}m$$

$$M_{bc} = \frac{EI_0}{L}\left(4\theta_a + 2\theta_b - 6\frac{\Delta_{ab}}{L}\right) + M^f_{bc}$$ (θ_a: 0; $\frac{\Delta_{ab}}{L}$: 0)

$$= \frac{EI_0}{6m}(4\theta_b) + M^f_{bc}$$

$$= \frac{2EI_0}{3m}\theta_b - 88.89\,kN\text{-}m$$

$$M_{cb} = \frac{EI_0}{6m}(2\theta_b) + M^f_{cb}$$

$$= \frac{EI_0}{3m}\theta_b + 71.11\,kN\text{-}m$$

Equilibrium

$$M_{ba} + M_{bc} = 0$$

$$\frac{9EI_o}{16m}\theta_b + 180\,kN\text{-}m + \frac{2EI_o}{3m}\theta_b - 88.89\,kN\text{-}m = 0$$

Solve

$$\frac{59EI_o}{48m}\theta_b = -91.11\,kN\text{-}m$$

$$\theta_b = -\frac{74.12\,kN\text{-}m^2}{EI_o} \quad (\circlearrowleft)$$

Back Sub.

$$M_{ab} = 0$$

$$M_{ba} = \frac{9}{16m} \times -74.12\,kN\text{-}m^2 + 180\,kN\text{-}m$$

$$= +138.3\,kN\text{-}m \quad (\therefore -)$$

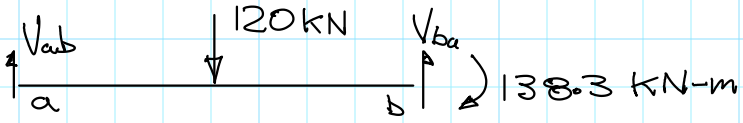
$$M_{bc} = \frac{2}{3m} \times -74.12\,kN\text{-}m^2 - 88.89\,kN\text{-}m$$

$$= -138.3\,kN\text{-}m \quad (\therefore \circlearrowright)$$

$$M_{cb} = \frac{1}{3m} \times -74.12\,kN\text{-}m^2 + 71.11\,kN\text{-}m$$

$$= +46.40\,kNm \quad (\therefore -)$$

Member end shears

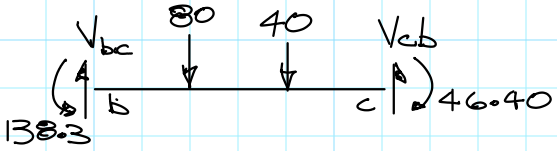


$\Sigma M_b = 0$ (+↺)

$-V_{ab}(8) + 120(4) - 138.3 = 0$

$V_{ab} = 42.71$ kN

$V_{ba} = 120 - 42.71 = 77.29$ kN



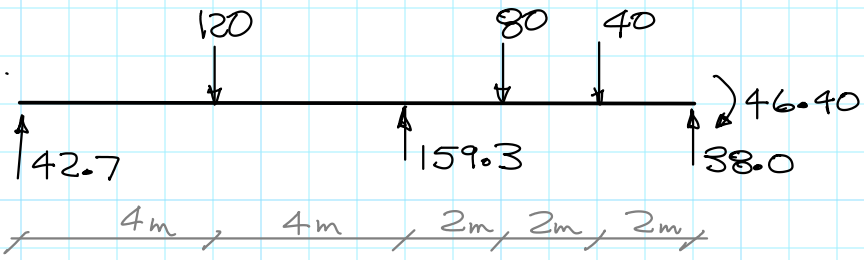
$V_{bc} = \frac{1}{6}(138.3 - 46.40 + 80\times4 + 40\times2)$ (ΣM_c)

$= 81.98$ kN

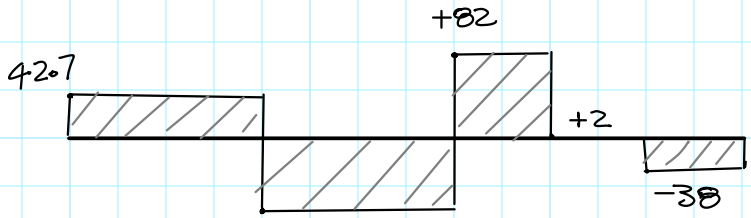
$V_{cb} = 80 + 40 - 81.98 = 38.02$

Reactions

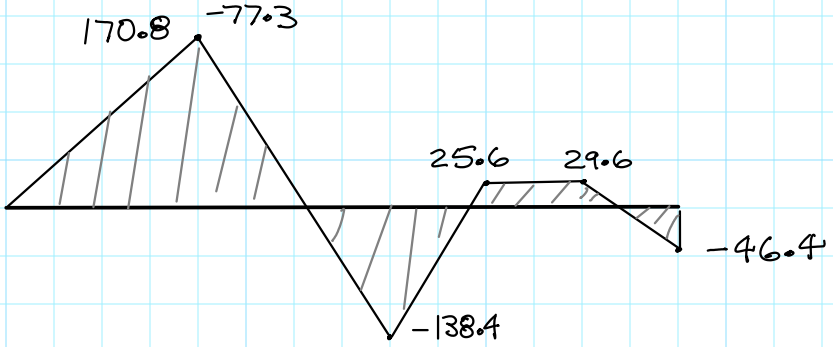
$R_b = 159.3$ kN



FBD



V
[kN]



M
[kN-m]

Note – V values come from integrating FBD & M values come from integrating V diag – this is an equilibrium check on whole beam.

OK.