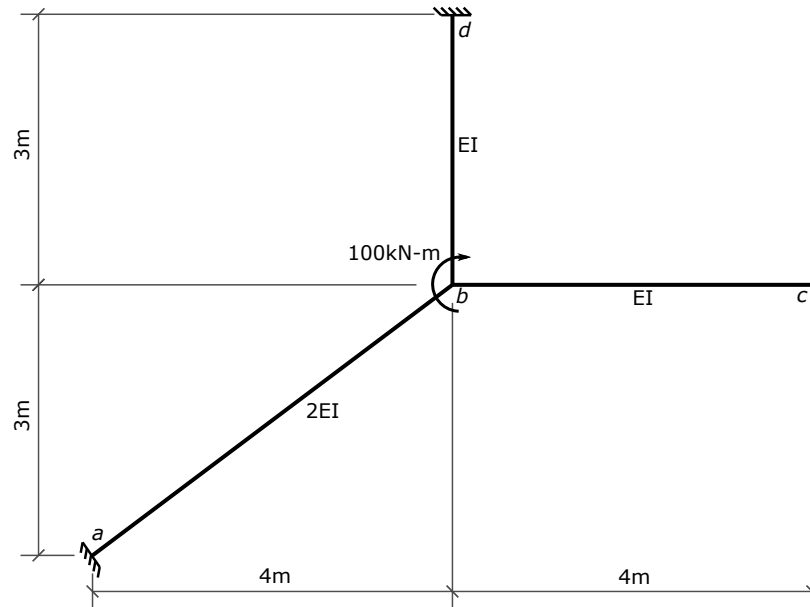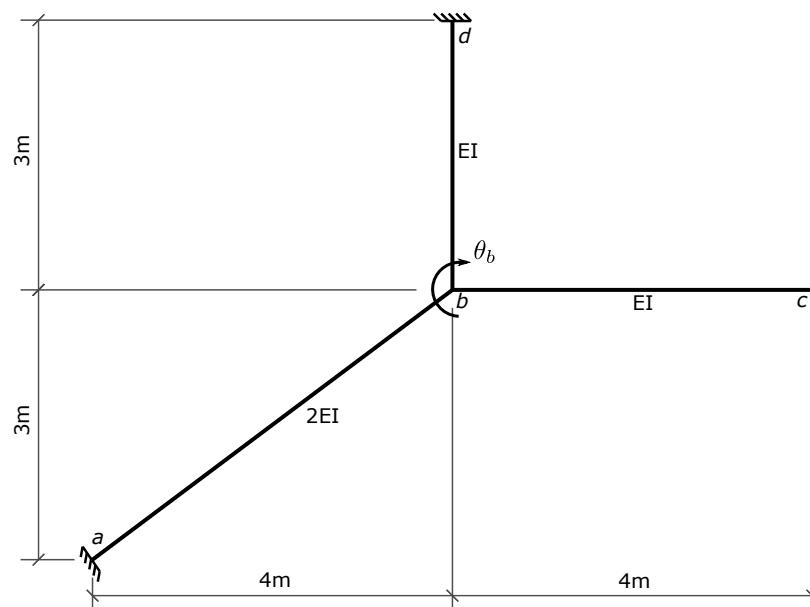# Problem 10 - Solution

1. Identify DOFs

There is 1 - θ_b, the rotation of joint b.

2. Fixed-end moments

There are no transverse loads on any of the 3 members, so all 6 member fixed end moments are zero.

```
In [1]:  Mfab = Mfba = Mfbc = Mfcb = Mfbd = Mfdb = 0
```

3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation, θ_b.

```
In [2]:  from sympy import symbols, solve, init_printing
         init_printing()
```

```
In [3]:  theta_b, EI = symbols('theta_b EI')
         theta_a = theta_c = theta_d = 0      # rotations at the outsaide end
          of each member
```

```
In [4]:  Mab = (2*EI/5)*(4*theta_a + 2*theta_b) + Mfab
         Mba = (2*EI/5)*(2*theta_a + 4*theta_b) + Mfba
         display(Mab,Mba)
```

$$\frac{4EI\theta_b}{5}$$

$$\frac{8EI\theta_b}{5}$$

```
In [5]:  Mbc = (EI/4)*(4*theta_b + 2*theta_c) + Mfbc
         Mcb = (EI/4)*(2*theta_b + 4*theta_c) + Mfcb
         display(Mbc,Mcb)
```

$$EI\theta_b$$

$$\frac{EI\theta_b}{2}$$

```
In [6]:  Mbd = (EI/3)*(4*theta_b + 2*theta_d) + Mfbd
         Mdb = (EI/3)*(2*theta_b + 4*theta_d) + Mfdb
         display(Mbd,Mdb)
```

$$\frac{4EI\theta_b}{3}$$

$$\frac{2EI\theta_b}{3}$$

4. Equilibrium Equation

The sum of the moments acting on joint b must be zero.

Note that the negatives of the member end forces act on the joint.

```
In [7]:  ee = (Mba + Mbc + Mbd) - 100   # = 0, +ive ccw on joint
         ee
```

Out[7]: $\dfrac{59EI\theta_b}{15} - 100$

5. Solve for displacement

```
In [8]:  ans = solve([ee],theta_b)
         ans
```

Out[8]: $\left\{ \theta_b : \dfrac{1500}{59EI} \right\}$

6. Back-substitute to get member end momentsa

```
In [9]:  mab = Mab.subs(ans).n()
         mba = Mba.subs(ans).n()
         display(mba, mab)
```

40.6779661016949

20.3389830508475

```
In [10]:  mbc = Mbc.subs(ans).n()
          mcb = Mcb.subs(ans).n()
          display(mbc,mcb)
```

25.4237288135593

12.7118644067797

```
In [11]:  mbd = Mbd.subs(ans).n()
          mdb = Mdb.subs(ans).n()
          display(mbd,mdb)
```

33.8983050847458

16.9491525423729

7. Check joint equilibrium

```
In [12]:  # sum of moments acting on joint, +ive ccw
          mba+mbc+mbd-100
```

Out[12]: 0

8. Member end shears

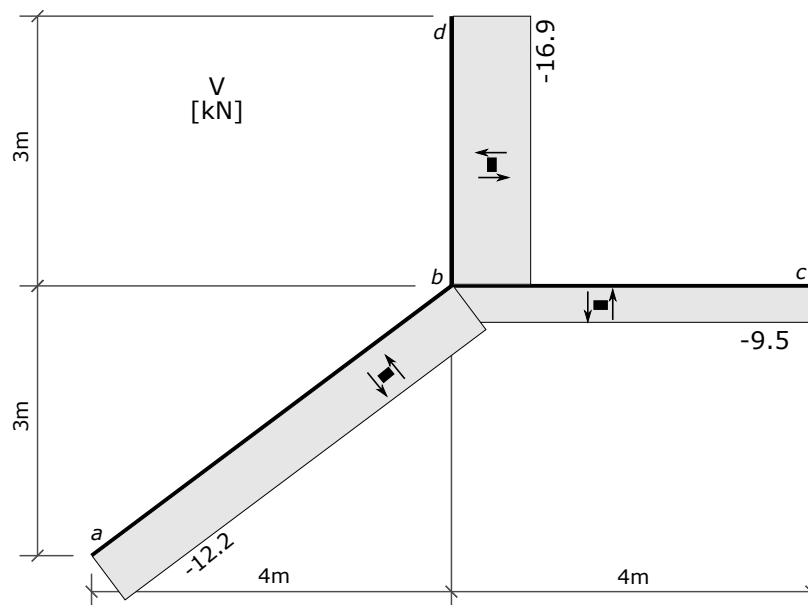As there are no transverse loads, the shears are constant (non-changing) over the whole length of each member.

```
In [13]:  vab = -(mab + mba)/5
          vbc = -(mbc + mcb)/4
          vbd = -(mbd + mdb)/3
          display(vab,vbc,vbd)
```
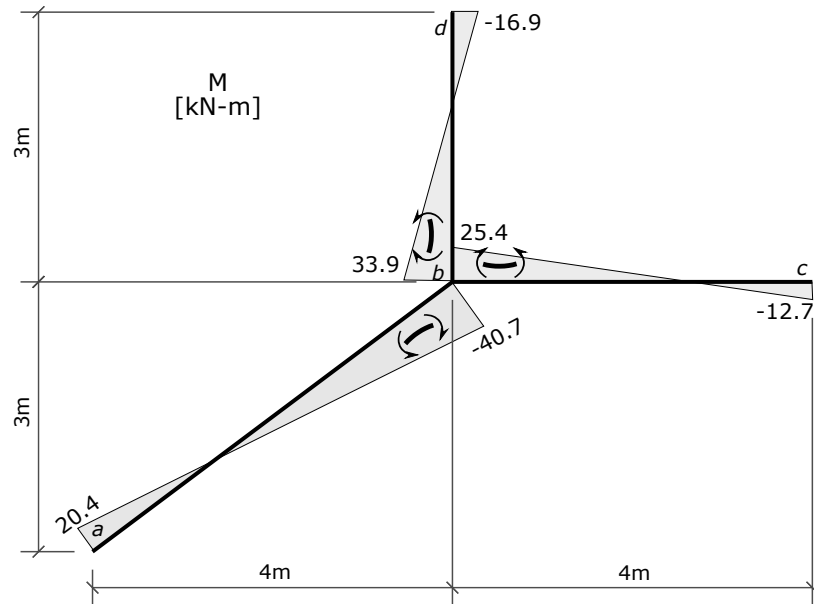
-12.2033898305085

-9.53389830508475

-16.9491525423729

9. Shear force diagram

10. Bending moment diagram

In [ ]: [   ]