Problem 11 - Solution
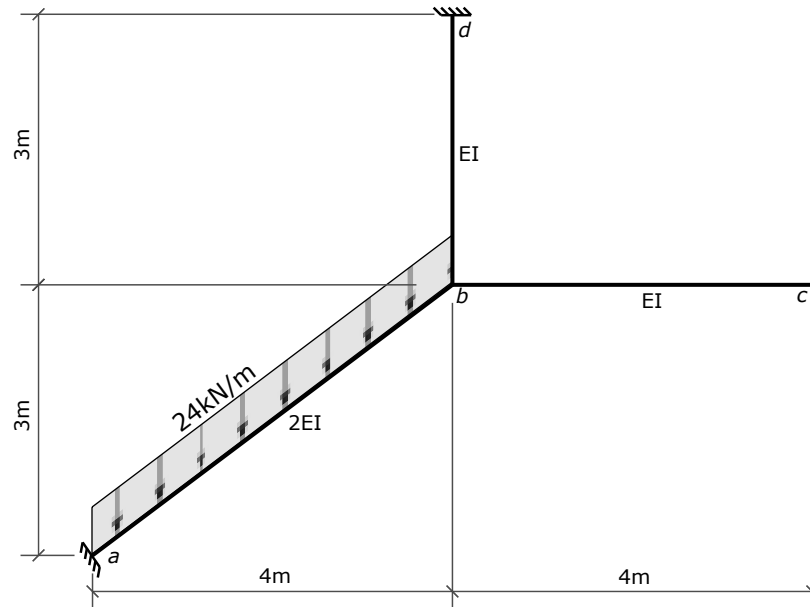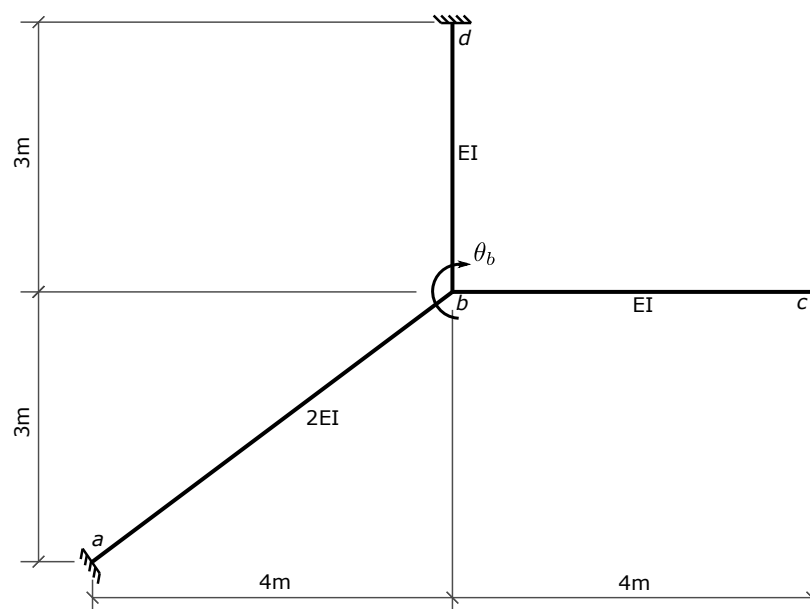

1. Identify DOFs

There is 1 - θ_b, the rotation of joint b.

2. Fixed-end moments

The distributed load on member ab must be resolved into its perpendicular component on ab. In this case, the perpendicular component is $0.8 \times 24 = 19.2$ kN/m.

There are no transverse loads on the two other members, so all 4 member fixed end moments for those are zero.

```
In [1]: Mfab = -0.8*24*5*5/12
        Mfba = 0.8*24*5*5/12
        Mfbc = Mfcb = Mfbd = Mfdb = 0
```

3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation, θ_b.

```
In [2]: from sympy import symbols, solve, init_printing
        init_printing()
```

```
In [3]: theta_b, EI = symbols('theta_b EI')
        theta_a = theta_c = theta_d = 0      # rotations at the outsaide end
         of each member
```

```
In [4]: Mab = (2*EI/5)*(4*theta_a + 2*theta_b) + Mfab
        Mba = (2*EI/5)*(2*theta_a + 4*theta_b) + Mfba
        display(Mab,Mba)
```

$$\frac{4EI\theta_b}{5} - 40.0$$

$$\frac{8EI\theta_b}{5} + 40.0$$

```
In [5]: Mbc = (EI/4)*(4*theta_b + 2*theta_c) + Mfbc
        Mcb = (EI/4)*(2*theta_b + 4*theta_c) + Mfcb
        display(Mbc,Mcb)
```

$$EI\theta_b$$

$$\frac{EI\theta_b}{2}$$

```
In [6]: Mbd = (EI/3)*(4*theta_b + 2*theta_d) + Mfbd
        Mdb = (EI/3)*(2*theta_b + 4*theta_d) + Mfdb
        display(Mbd,Mdb)
```

$$\frac{4EI\theta_b}{3}$$

$$\frac{2EI\theta_b}{3}$$

4. Equilibrium Equation

The sum of the moments acting on joint b must be zero.

Note that the negatives of the member end forces act on the joint.

```
In [7]: ee = (Mba + Mbc + Mbd)   # = 0, +ive ccw on joint
        ee
```

Out[7]: $\dfrac{59EI\theta_b}{15} + 40.0$

5. Solve for displacement

```
In [8]: ans = solve([ee],theta_b)
        ans
```

Out[8]: $\left\{ \theta_b : -\dfrac{10.1694915254237}{EI} \right\}$

6. Back-substitute to get member end moments

```
In [9]: mab = Mab.subs(ans).n()
        mba = Mba.subs(ans).n()
        display(mba, mab)
```

23.728813559322

-48.135593220339

```
In [10]: mbc = Mbc.subs(ans).n()
         mcb = Mcb.subs(ans).n()
         display(mbc,mcb)
```

-10.1694915254237

-5.08474576271186

```
In [11]: mbd = Mbd.subs(ans).n()
         mdb = Mdb.subs(ans).n()
         display(mbd,mdb)
```

$$-13.5593220338983$$

$$-6.77966101694915$$

7. Check joint equilibrium

```
In [12]: # sum of moments acting on joint, +ive ccw
         mba+mbc+mbd
```

Out[12]: $8.88178419700125 \cdot 10^{-15}$

8. Member end shears

As there are no transverse loads and bc and bd, the shears on those are constant (non-changing) over the whole length of each member.

```
In [13]: vab = -(mab + mba - 0.8*24*5*5/2)/5
         vba = 0.8*24*5 - vab
         vbc = -(mbc + mcb)/4
         vbd = -(mbd + mdb)/3
         display(vab,vba, vbc,vbd)
```
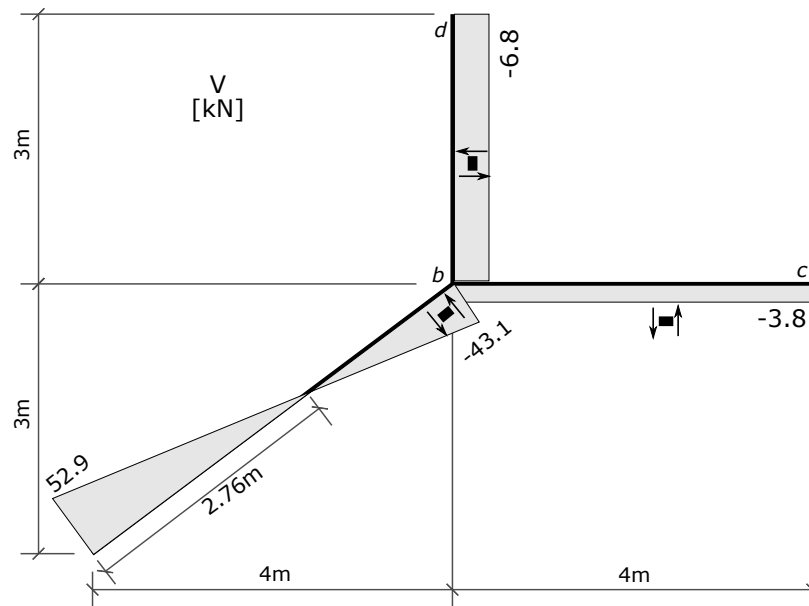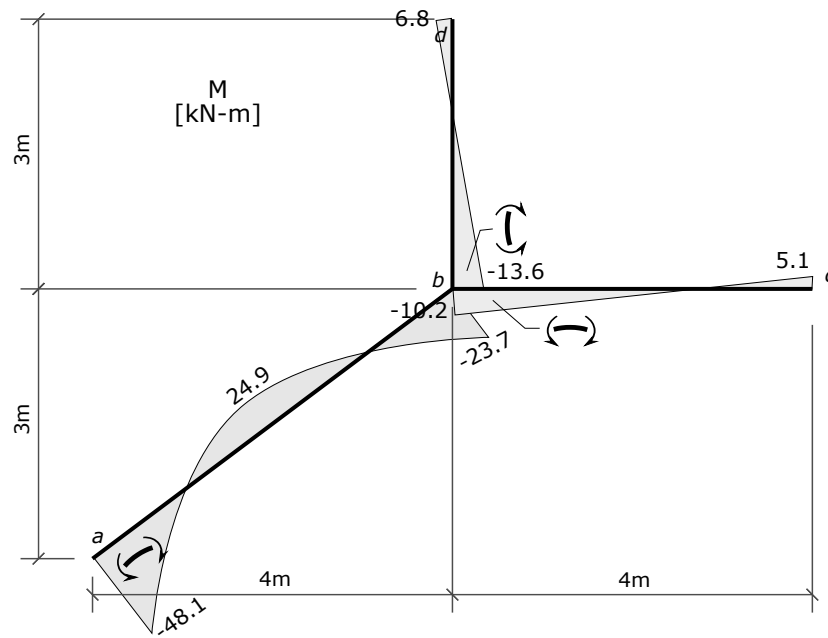
$$52.8813559322034$$

$$43.1186440677966$$

$$3.8135593220339$$

$$6.77966101694915$$

9. Shear force diagram

10. Bending moment diagram


```
In [ ]:
```