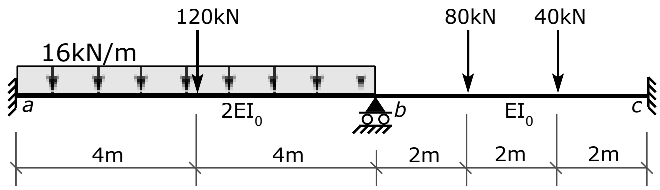
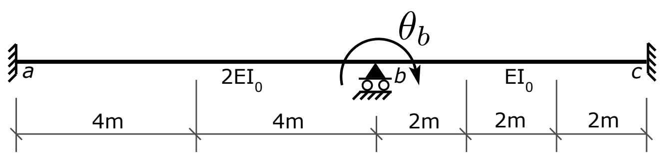
Problem 12 - Solution



1. Identify DOFs

There is one DOF - the rotation of joint b, θ_b.



2. Fixed-end moments

Each span, ab and bc, has multiple loads on it. We use the principle of superposition and sum the contributions of each load.

Member ab:

In [1]:
```
Mfab = -16*8**2/12 + (-120*8/8)
Mfba = 16*8**2/12 + 120*8/8
Mfab, Mfba
```

Out[1]: (-205.33333333333331, 205.33333333333331)

Member ba:

In [2]:
```
Mfbc = -80*2*4**2/6**2 + -40*4*2**2/6**2
Mfcb = 80*2**2*4/6**2 + 40*4**2*2/6**2
Mfbc, Mfcb
```

Out[2]: (-88.88888888888889, 71.11111111111111)

3. Slope deflection equations

Express member end moments as a function of the unknown joint rotation, θ_b.

In [3]:
```
from sympy import symbols, solve, init_printing
init_printing()
```

In [4]:
```
theta_b, EI = symbols('theta_b EI')
theta_a = theta_c = 0      # rotations at the outside ends are zero
 (fixed support)
```

In [5]:
```
Mab = (2*EI/8)*(4*theta_a + 2*theta_b) + Mfab
Mba = (2*EI/8)*(2*theta_a + 4*theta_b) + Mfba
display(Mab,Mba)
```

$$\frac{EI\theta_b}{2} - 205.333333333333$$

$$EI\theta_b + 205.333333333333$$

In [6]:
```
Mbc = (EI/6)*(4*theta_b + 2*theta_c) + Mfbc
Mcb = (EI/6)*(2*theta_b + 4*theta_c) + Mfcb
display(Mbc,Mcb)
```

$$\frac{2EI\theta_b}{3} - 88.8888888888889$$

$$\frac{EI\theta_b}{3} + 71.1111111111111$$

4. Equilibrium Equation

The sum of the moments acting on joint b must be zero.

Note that the negatives of the member end forces act on the joint.

In [7]:
```
ee = (Mba + Mbc)  # = 0, +ive ccw on joint
ee
```

Out[7]:

$$\frac{5EI\theta_b}{3} + 116.444444444444$$

5. Solve for displacement

In [8]:
```
ans = solve([ee],theta_b)
ans
```

Out[8]:
$$\left\{\theta_b : -\frac{69.8666666666664}{EI}\right\}$$

Therefore, the joint rotates counter-clockwise. This makes sense as the loads are greater on the left span, and that span is longer, with considerabley larger fixed end moments. So a counter clockwise rotation will serve to reduce the left hand side moments and increase the right hand side moments until they are balanced.

6. Back-substitute to get member end moments

In [9]:
```
mab = Mab.subs(ans)
mba = Mba.subs(ans)
display(mab, mba)
```

$$-240.266666666667$$

$$135.466666666667$$

In [10]:
```
mbc = Mbc.subs(ans)
mcb = Mcb.subs(ans)
display(mbc,mcb)
```

$$-135.466666666666$$

$$47.8222222222223$$

7. Check joint equilibrium

The sum should be zero or very close to it.

In [11]:
```
# sum of moments acting on joint, +ive ccw
mba+mbc
```

Out[11]: $4.54747350886464 \cdot 10^{-13}$

It is, so OK.

8. Member end shears

Member ab:

In [12]:
```
vab = -(mab + mba - 16*8*8/2 - 120*4)/8     # from sum M about b for member ab +ive CW
vba = 16*8 + 120 - vab                      # from sum Fy
display(vab,vba)
```

137.1

110.9

Member bc:

In [13]:
```
vbc = -(mbc + mcb - 80*4 - 40*2)/6          # from sum M about c for member bc, +ive CW
vcb = 80 + 40 - vbc                         # from sum Fy
display(vbc,vcb)
```

81.274074074074

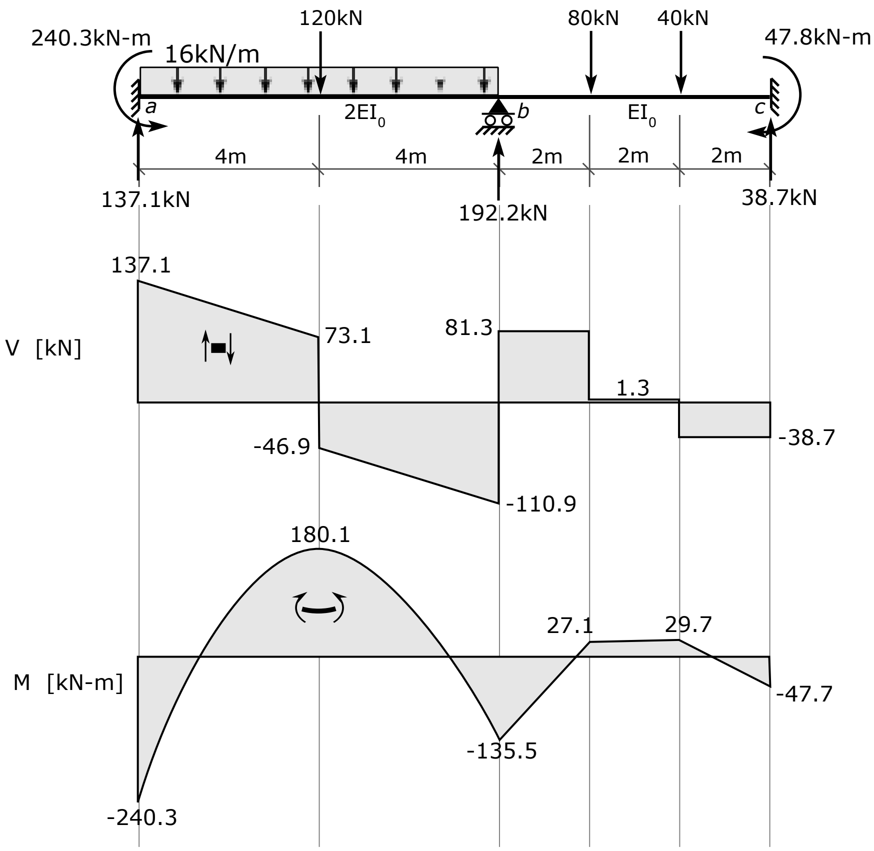
38.725925925926

and the reaction at b:

In [14]:
```
Vb = vba + vbc
Vb
```

Out[14]: 192.174074074074

9. Summary


240.3kN-m
16kN/m
120kN
80kN
40kN
47.8kN-m
a
2EI0
b
EI0
c
4m
4m
2m
2m
2m
137.1kN
192.2kN
38.7kN
137.1
V [kN]
73.1
81.3
1.3
-46.9
-38.7
-110.9
180.1
27.1
29.7
M [kN-m]
-47.7
-135.5
-240.3

In [ ]: