Principle of Virtual Displacements
(Bernoulli 1717)

If a rigid body is in equilibrium under a set of P forces and it is subjected to any virtual displacement, the virtual work done by the P forces is zero.

(West, 1993)

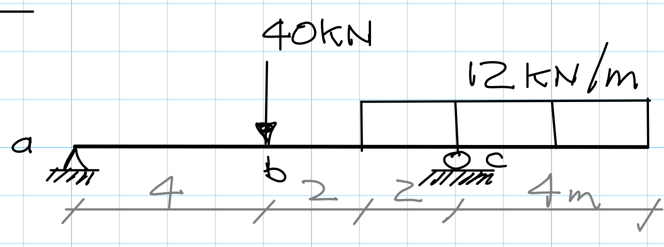
$$\sum \delta^{*} F = 0$$

F ← real

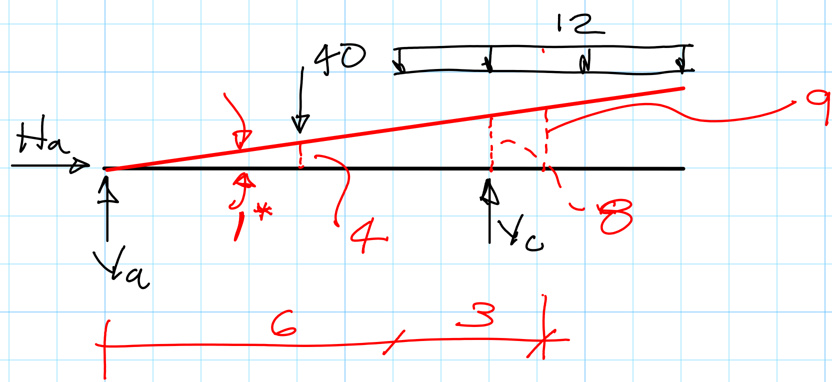
δ^{*} ← virtual

[This agrees with virtual work principal where external work = internal strain energy. Strain energy = 0 for a rigid structure because strain = 0]

Example 1



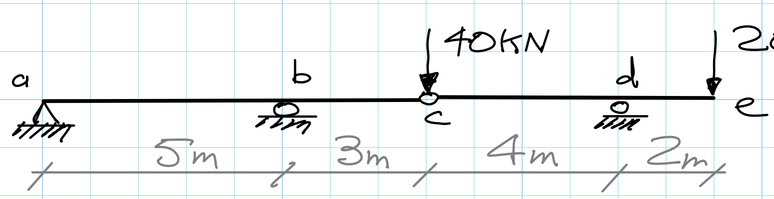
Compute vertical reaction at pt. c.

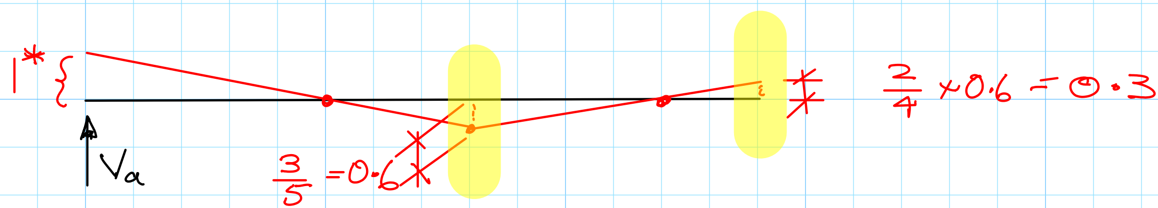


$$-40 \times 4 + V_c \times 8 - 12 \times 6 \times 9 = 0$$

$$V_c = 101 \text{ kN}$$

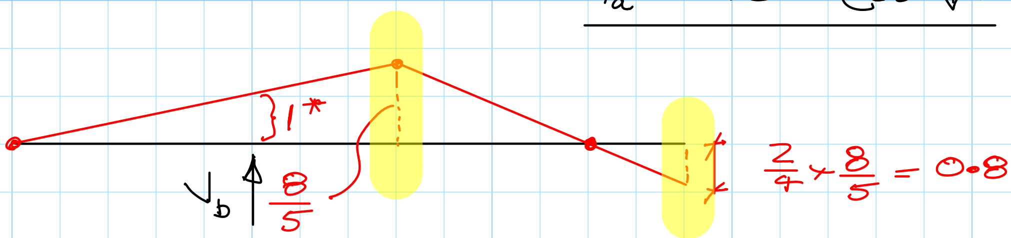
Example 2 - Compute reactions @ a, b, d





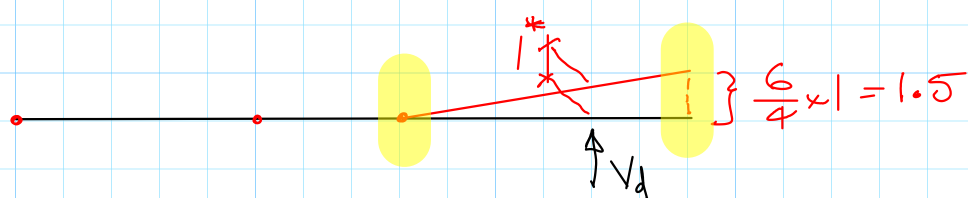
$$V_a(1) + 40 \times 0.6 - 20 \times 0.3 = 0$$

$$V_a = -18 \quad (\therefore \downarrow)$$



$$V_b(1) - 40(1.6) + 20 \times 0.8 = 0$$

$$V_b = 48 \quad (\therefore \uparrow)$$

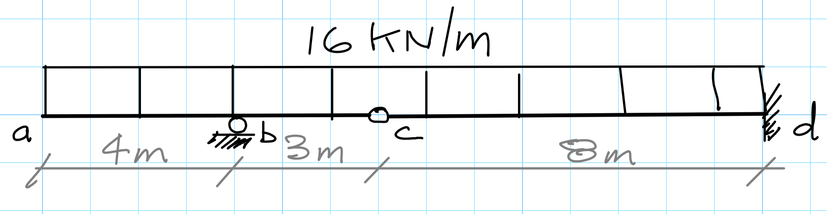


$$V_d(1) - 20 \times 1.5 = 0$$

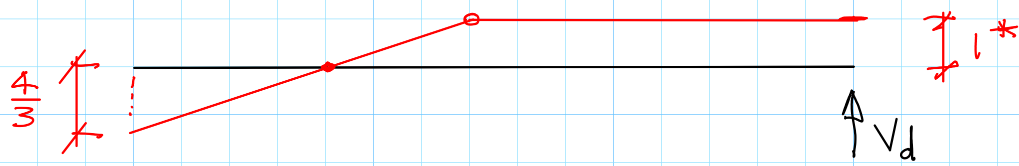
$$V_d = 30 \quad (\therefore \uparrow)$$

Example 3

- vert reaction & moment @ d

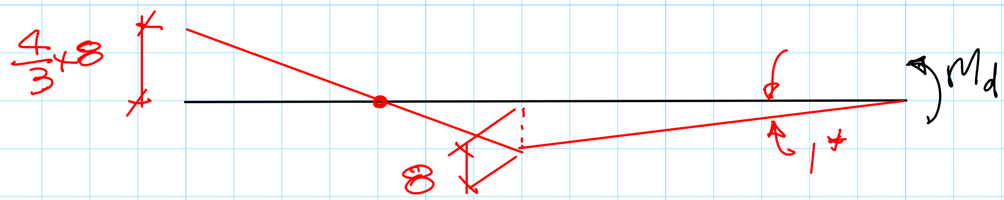


(work done by uniform dist load = total load x distance moved by centroid)



$$V_d(1) - 16\times8\times1 - 16\times3\times1\times\frac{1}{2} + 16\times4\times\frac{4}{3}\times\frac{1}{2} = 0$$

$$\underline{V_d = 109.3 \text{ kN } (\downarrow)}$$ ←



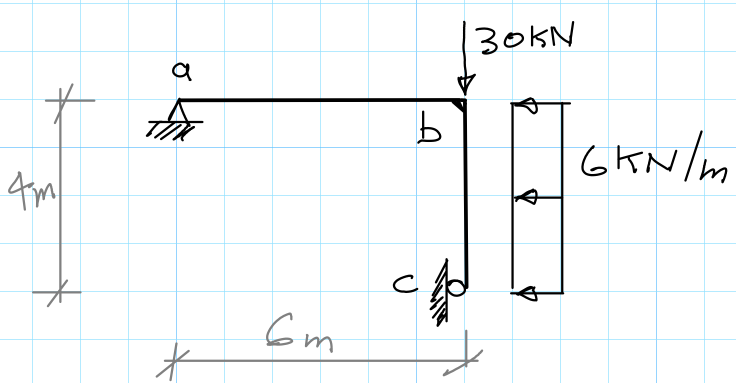
$$M_d(1) + 16\times8\times8\times\frac{1}{2} + 16\times3\times8\times\frac{1}{2} - 16\times4\times\frac{4}{3}\times8\times\frac{1}{2} = 0$$

$$\underline{M_d = -362.7 \ (\therefore \circlearrowleft)}$$ ←

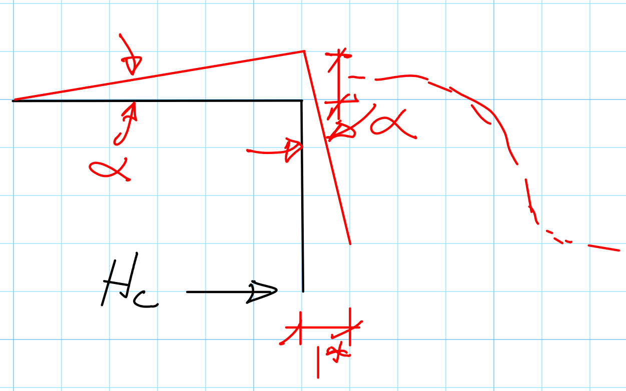
Example 4

Find horizontal reaction @ pt. C

revised Nov. 8/10



Virtual horizontal displacement of pt. c.



$4\alpha = 1$

$\alpha = 1/4$ } from bc

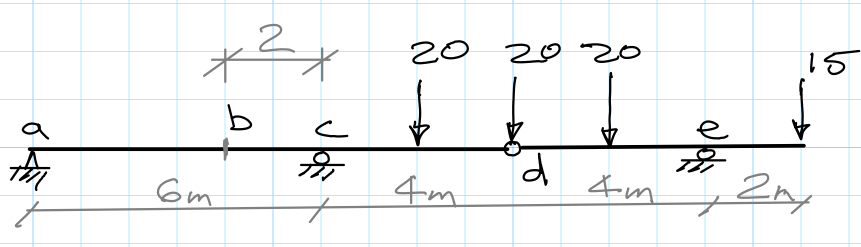
$6\alpha = \frac{6}{4} = 1.5$ - from ab

(all portions rotate equally)

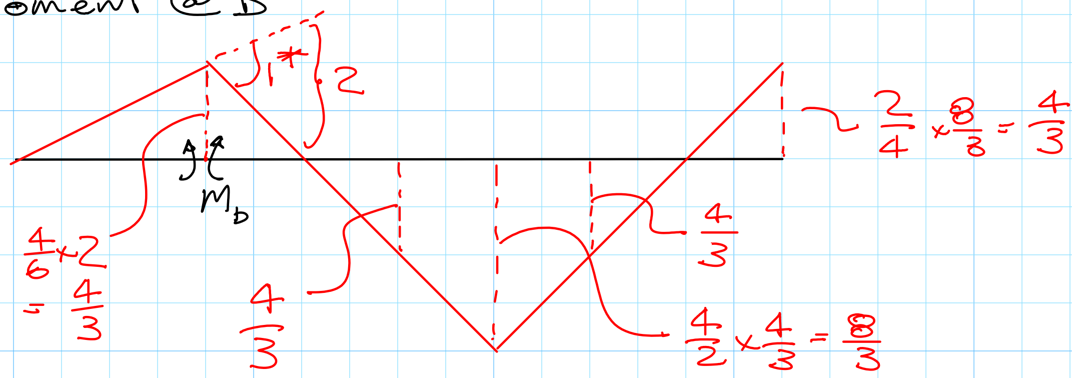
$$H_c(1) - 6 \times 4 \times 1 \times \tfrac{1}{2} - 30 \times 1.5 = 0$$

$$H_c = 57 \ (\rightarrow)$$

Example 5 – Internal Forces



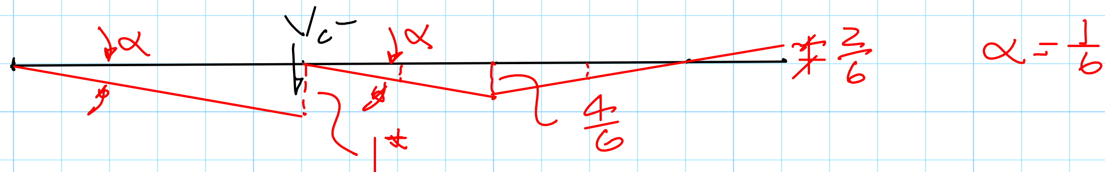
a) moment @ b



$$M_b(1) + 20\times\frac{4}{3} + 20\times\frac{8}{3} + 20\times\frac{4}{3} - 15\times\frac{4}{3} = 0$$

$$M_b = -86.7$$ (∴ ...)

b) shear left of c



$$V_{c^-}(1) + 20\times\frac{2}{6} + 20\times\frac{4}{6} + 20\times\frac{2}{6} - 15\times\frac{2}{6} = 0$$

$$V_c = -21.7$$ (∴ ...)

c) shear right of pin @ d

$$V_{d^+}(1) + 20\times\frac{1}{2} - 15\times\frac{1}{2}$$

$$V_d = -2.5$$ (∴ ...)