CIVE 3203

Displacements due to Shear by Virtual Work

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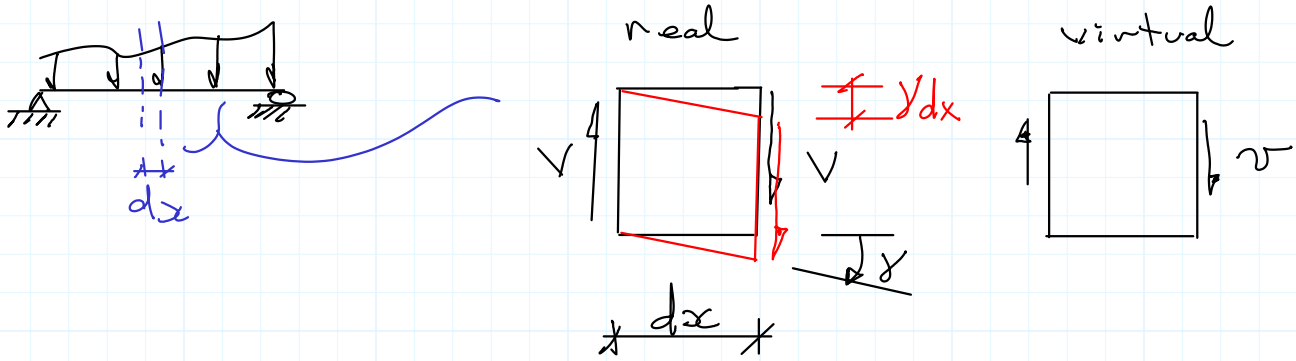
Revision History:
1. Nov. 13, 2012 - original posting.

VW Relationship: $1 \cdot \Delta = \sum \int v \, dL$

$1 \cdot \Delta$: external work (v virtual)

$\sum \int v \, dL$: strain energy (v virtual)

Strain Energy – Transverse Shear



$\gamma = \frac{\tau}{G}$ $\quad \tau$ = avg. shear stress across cross-section

$\tau \approx k\left(\frac{V}{A}\right)$

V – applied shear force
A – cross-sect. area
k – form factor
1.2 rect. x-sect
≈1.0 for WF (A = hw of web)

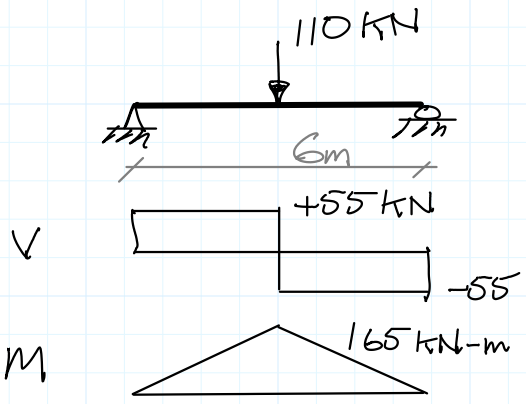
element strain energy $= v \gamma \, dx$

$= v \frac{k\left(\frac{V}{A}\right)}{G} dx$

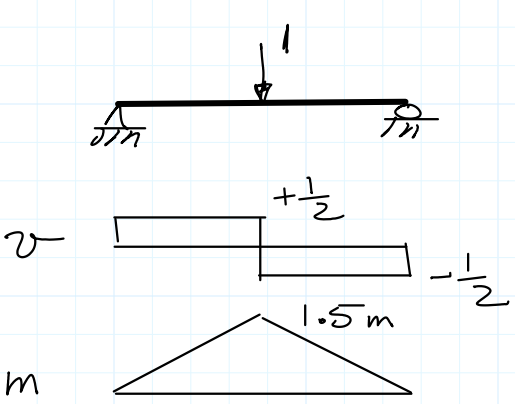
$= k \frac{vV}{GA} dx$

total shear s.e. $= \int_{\text{length}} k \frac{vV}{GA} dx$

Real

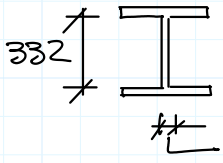


Virtual



Beam

W360×33



$I = 82.6 \times 10^6 \text{ mm}^4$

$A_w = 332 \times 5.8 = 1930 \text{ mm}^2$ $\quad k = 1.0$

$E = 200\,000 \text{ MPa}$

$G = 77\,000 \text{ MPa}$

Defln due to bending

$$1 \times \Delta_b = \frac{2}{EI} \int \; \overset{1500}{\underset{3000}{\triangle}} \; \overset{165\times10^6}{\triangle} \qquad \text{(units of N, mm)}$$

$$= \frac{2}{200\,000 \times 82.6 \times 10^6} \times \frac{3000}{3} \times 1500 \times 165 \times 10^6$$

$$= 89.9 \text{ mm}$$

Defln due to shear

$$1 \times \Delta_v = 2 \times \frac{k}{GA} \int \; \overset{0.5}{\underset{3000}{\square}} \; \overset{55000}{\underset{3000}{\square}}$$

$$= 2 \times \frac{1}{77\,000 \times 1930} \times 3000 \times 0.5 \times 55000$$

$$= 1.1 \text{ mm}$$

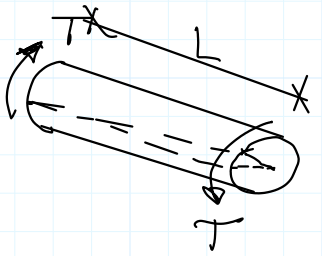
Total Defln $= 89.9 + 1.1 = 91.0$ mm

(1.2 % due to shear)

(usually negligible in most structures)

S.E Torsion

Circular Shafts



Total S.E. in shaft:

$$U_t = \frac{tTL}{GJ}$$

J – polar moment of inertia $= \frac{\pi r^4}{2}$

Summary

Virtual S.E.

axial loads $\frac{nNL}{AE}$

torsion (circular shaft) $\frac{tTL}{GJ}$

beam (flexure) $\int_{length} \frac{mM}{EI}\,dx$

beam (shear) $\int_{length} k\frac{vV}{GA}\,dx$

(k = form factor, $1.0 \le k \le 1.2$)

These effects must be combined, where appropriate, in the right side of

$$1(\Delta) = \sum udL$$

$$= \sum \text{strain energy (virtual)}$$

Chapter 9 (in text)

differential temp (p 372) not covered
(though temp changes in trusses is)

9-7 Castigliano's Theorem – not covered.