CIVE 3205

Example C60

Eccentrically Loaded Bolts

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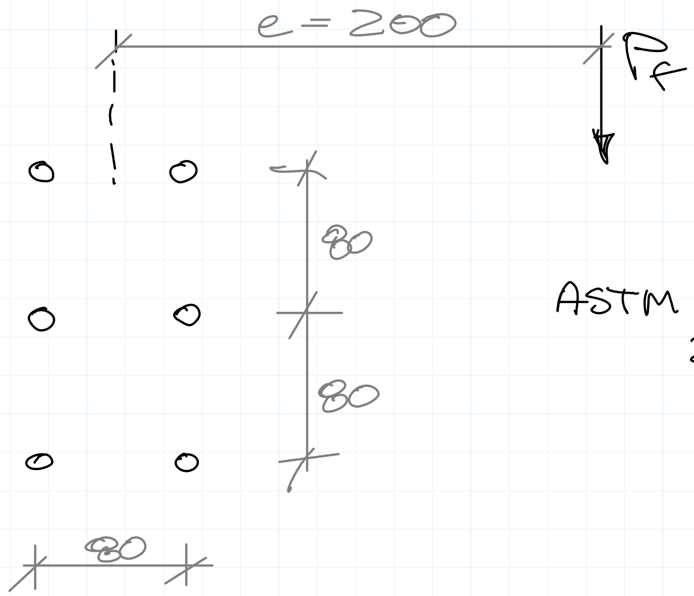
Revisions:

- Feb 26/20: new posting.

Compute the capacity, P_n, of the eccentrically loaded bolt group.



ASTM A325M bolts
20M

Force in a single bolt

$$R = R_u\left(1 - e^{-\mu\Delta}\right)^{\lambda}$$

R_u = ult. shearing force

= 329 kN (unfactored double shear threads excluded)

$\mu = 0.394$

$\lambda = 0.55$

Δ = shearing deformation in bolt, mm.

Δ is proportional to distance from instant centre of rotation

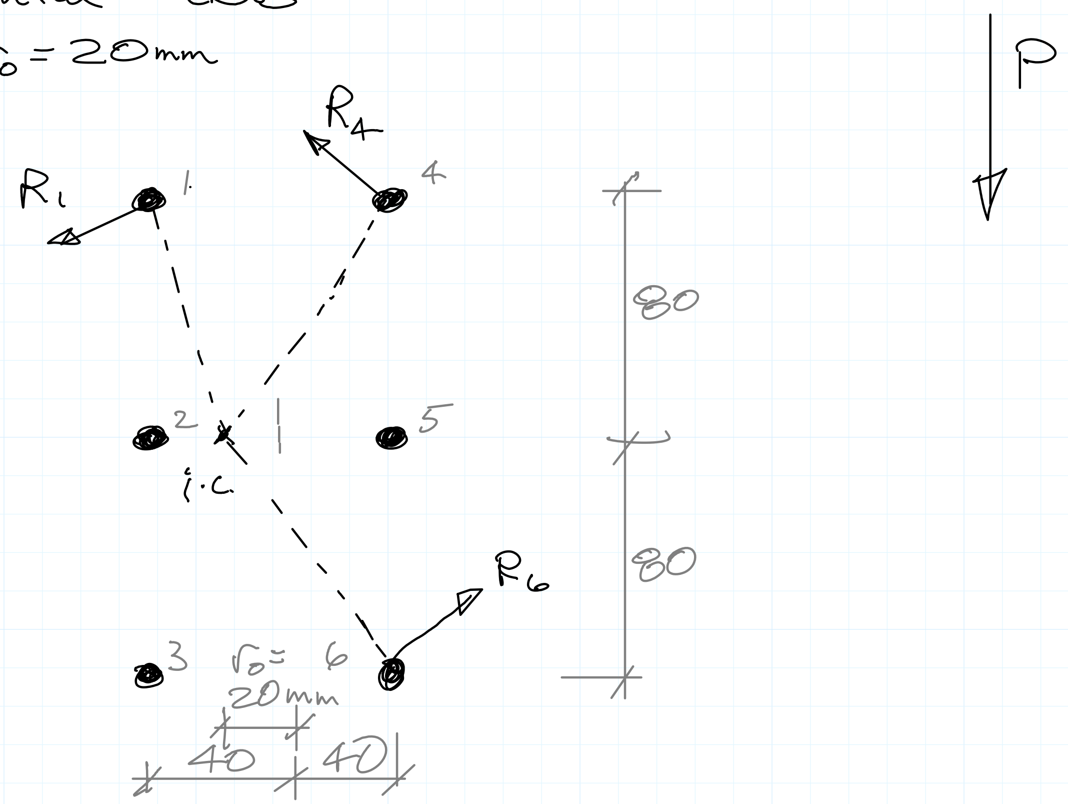
Bolt furthest from i.c. is assummed to be at its ultimate deformation.

$$\Delta_u = 8.64 \text{ mm}$$

Use r_o = dist leftward of i.c from centroid

Choose a position of i.c. on the horizontal axis

try $r_o = 20$mm



Assume that one plate rotates rigidly about i.c. Then shear deformation in each bolt is proportional to distance from i.c.

Bolts 4 & 6 are furthest from i.c.

$$r_{max} = r_4 = r_6 = \sqrt{60^2 + 80^2} = 100 \text{ mm}$$

The deformation of 4 & 6 is

$\Delta_4 = \Delta_6 = \Delta_v = 8.64\,mm$ ✓

The force in 4 & 6 is

$$R_4 = R_6 = 329\left(1 - e^{-0.394 \times 8.64}\right)^{.55}$$

$$= 322.9 \text{ kN}$$

Bolts 1 & 3 – distance from i.c.

$$r_1 = r_3 = \sqrt{20^2 + 80^2} = 82.46 \text{ mm}$$

shear deformations are:

$$\Delta_i = \frac{r_i}{r_{max}} \Delta_{max}$$

$$r_1 = r_3 = \frac{82.46}{100} \times 8.64$$

$$= 7.125 \text{ mm}$$

the forces in bolts 1 & 3

$$R_1 = R_3 = 329\left(1 - e^{-.394 \times 7.125}\right)^{.55}$$

$$= 317.9 \text{ kN}$$

Bolt 2:

$$r_2 = 20 \text{ mm}$$

$$\Delta_2 = \frac{20}{100} \times 8.64 \text{ mm} = 1.728 \text{ mm}$$

$$R_2 = 329\left(1 - e^{-.394 \times 1.728}\right)^{.55}$$

$$= 223.2 \text{ kN}$$

Bolt 5:

$$r_5 = 60 \text{mm}$$

$$\Delta_5 = \frac{60}{100} \times 8.64 \text{mm} = 5.184 \text{ mm.}$$

$$R_5 = 329\left(1 - e^{-.394 \times 5.184}\right)^{.55}$$

$$= 304.8 \text{ kN.}$$

Now, each of these forces has a vertical component, V_i

$$V_i = \frac{x_i}{r_i} R_i$$

& produces a moment about the i.c., M_i

$$M_i = r_i R_i$$

Bolts 4&6

$$V_4 = V_6 = \frac{60}{100} \times 322.9 = 193.7 \text{ kN}$$

$$M_4 = M_6 = 100 \times 322.9 = 32290 \text{ kN-mm}$$

Bolts 1&3

$$V_1 = V_3 = \frac{-20}{82.46} \times 317.9 = -77.10 \text{ kN}$$

$$M_1 = M_3 = 82.46 \times 317.9 = 26214 \text{ kN-mm}$$

Bolt 2

$$V_2 = \frac{-20}{20} \times 223.2 = -223.2 \text{ kN}$$

$$M_2 = 20 \times 223.2 = 4464 \text{ kN-mm}$$

Bolt 5

$$V_2 = \frac{60}{60} \times 304.8 = 304.8 \text{ kN}$$

$$M_2 = 60 \times 304.8 = 18288 \text{ kN-mm}$$

If the i.c. is at the correct place, these forces will be in equilibrium.

First, we can calculate the value of P causing this failure from

$$P_u (e + r_o) = \Sigma M_i$$

$$P_u (200+20) = 32290 \times 2 + 26214 \times 2 + 4464 + 18288$$

$$P_u = 635.3 \text{ kN}$$

this is the ultimate load that is in equilibrium with the moment caused by the bolt forces.

Now check $\sum F_y \ +\uparrow$

$$\sum F_y = P_u + \sum V_i$$

$$= -635.3 + 193.7 \times 2 + -77.10 \times 2 + -223.2 + 304.8$$

$$= -320.5 \qquad \neq 0 \qquad \text{N.G.}$$

The ult load, P_u, should also be in equilibrium with the vertical components of the bolt forces.

In this case it isn't.

$\therefore$ The position chosen for $r_o = 20$mm is not correct.

This procedure must be repeated with different values of r_o until

$$P_u = \frac{\sum M_i}{e + r_o} = \sum V_i$$

For this problem, that happens (closely enough) at:

$r_o = 35.68$mm

$\sum M_i = 145.3 \times 10^3$ kN-mm

$\sum V_i = 616$ kN

$$\underline{\underline{P_u = 617 \text{ kN}}} \leftarrow$$

In tabular form, the calculations are:

$r_0 = 35.7$ mm

bolt #	x_i (mm)	y_i (mm)	r_i (mm)	$\Delta_i = \frac{r_i}{r_{max}} \Delta_{max}$	R_i (kN)	V_i (kN)	M_i (kN-mm)
1	−4.3	80	80.12	6.286	313	−17	25114
2	−4.3	0	4.32	0.339	105	−105	453
3	−4.3	−80	80.12	6.286	313	−17	25114
4	75.7	80	110.12	8.640	323	222	35561
5	75.7	0	75.68	5.937	311	311	23545
6	75.7	80	110.12	8.640	323	222	35561
					Σ	616	145.3×10^3

$$P_u = \frac{145.3 \times 10^3}{35.7 + 200} = 616 \text{ kN}$$

Equilibrium satisfied

$P_u = \Sigma V_i$

$\therefore r_0$ is correct

$\therefore P_u = 616$ kN

The force R_u for a single bolt is unfactored, double shear A325M M20 bolts, threads excluded.

$P_f \leq \phi_b P_u \leq 0.8 \times 616 \times \frac{1}{2}$

$\underline{P_f \leq 246 \text{ kN}}$ single shear

The above is not practical for hand calculation.

Tables 3-14 thru 3-20 are design aids for this.

From Table 3-15, 2 rows of bolts

pitch = 80 mm 3 bolts/row

eccentricity = 200mm

$C = 1.91$

$V_r = 125\,kN$ (factored, single shear, threads excluded)

$P_f \le C V_r$

$\le 1.91 \times 125\,kN$ (threads excluded)

$\underline{\underline{P_f \le 239\,kN}}$ ←

(cf 246 kN, above)
(within 3%)

Must ensure threads excluded, otherwise reduce strength to 0.70

Must ensure that bearing resistance is greater than shear resistance of 125 kN/bolt.